MATHEMATICS 8

Number Applications

Distance Learning

Module 3

Alberta EDUCATION

Mathematics 8

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Mathematics 8
Student Module Booklet
Module 3
Number Applications
Learning Technologies Branch
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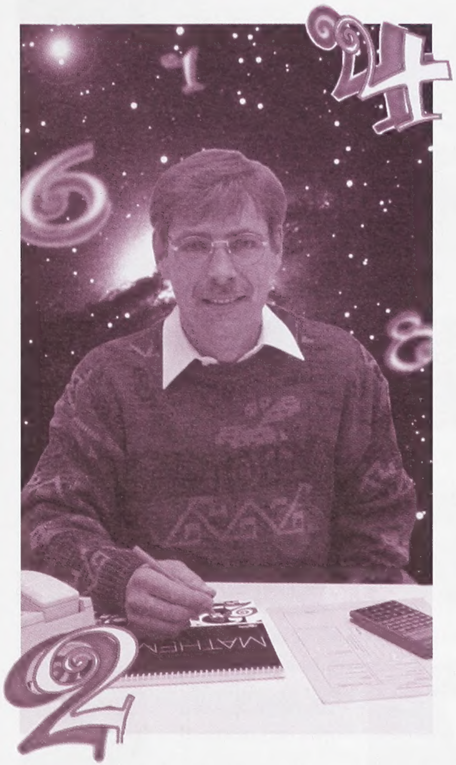
This document is intended for	
Students	✓
Teachers	✓
Administrators	
Parents	
General Public	
Other	

The Learning Technologies Branch has an Internet site that you may find useful. The address is as follows:

http://ednet.edc.gov.ab.ca/ltb

The use of the Internet is optional. Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

Welcome



Mathematics 8 contains six modules. Work through the modules in the order given, since several concepts build on each other as you progress in the course.

Mathematics 8

Module 1
Number
Connections

Module 2
Patterns and
Relations

Module 3
Number
Applications

Module 4
Two-Dimensional
Geometry

Module 5
Statistics and
Probability

Module 6
Three-Dimensional
Geometry

Welcome to Module 3. We hope you'll enjoy your study of Number Applications.

The document you are presently reading is called a Student Module Booklet. You may find visual cues or icons throughout it. Read the following explanations to discover what each icon prompts you to do.

- Prepare for a problem that will provide a change of topic.

- Prepare for a challenging problem related to the topic of the activity.

- Use the Internet to explore a topic.

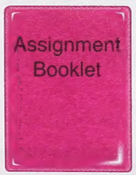
- Use computer software.

- Use a scientific calculator.

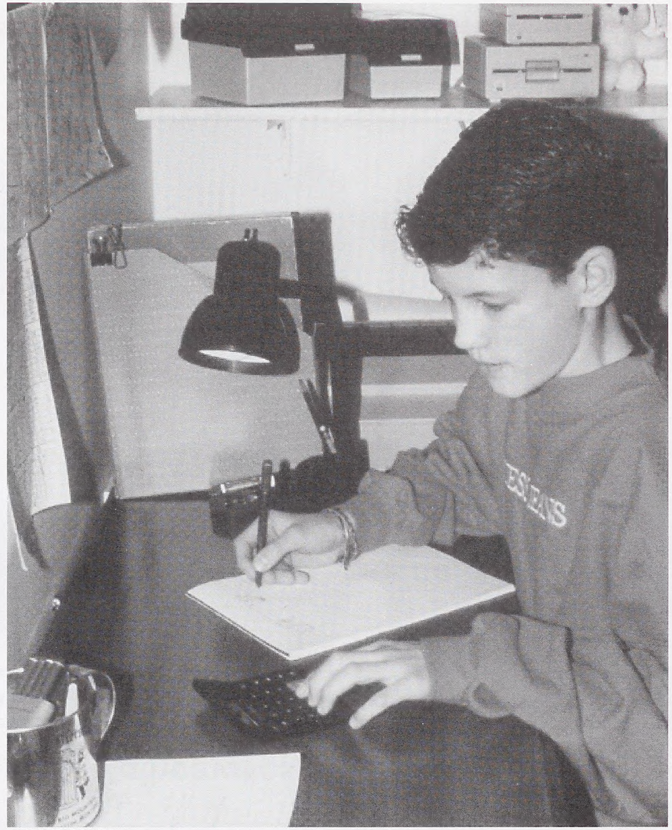
- View a videocassette.

- Pay close attention to important words or ideas.

- Use the suggested answers in the Appendix to correct activities.



- Answer the questions in the Assignment Booklet.



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There are no response spaces provided in this Student Module Booklet. This means that you will need to use your own paper for your responses. You should keep your response pages in a binder so that you can refer to them when you are reviewing or studying.

Technology

Today society is turning to **technology** more than ever before, and it is to your advantage to be able to effectively use technology when required.

Technology is the application of tools, materials, and processes to the solution of problems. More specifically, technology refers to devices and systems that are used in processing, transferring, storing, and communicating information through electronic media.

In Mathematics 8, along with the course materials, you will use a calculator, computer, and videocassette player as tools for learning and doing mathematics.

Calculators are helpful tools for solving problems and exploring patterns and relationships between numbers. Using a calculator will also save you time and help you develop your estimating skills. Therefore, you will be given numerous opportunities in each module to use a calculator.

Computers are useful for organizing and displaying data, or drawing figures. For this reason you will have the chance in many activities to work with popular computer applications such as spreadsheets and draw programs. You will also want to check out the many Internet connections in each module.

Videocassette players allow you to view video programs on key concepts that are difficult to explain in print. That is why video programs are cited in this course.

It is expected that all of you will be able to view the video programs and use a calculator, and that most of you will do the computer activities. However, if you are unable to access a computer, you may do the calculations using a calculator, and draw figures and graphs by hand.

Problem-Solving Skills

One of the exciting features of this course is that you will develop and improve your ability in problem solving. You will need these problem-solving skills many times in your lifetime. Since this course focuses on problem solving, it is important that you understand what a **problem** is.

A problem is a task for which the method of finding the answer (as well as the answer) is not immediately known.

Like any skill, the skill of problem solving must be developed. Problems may or may not involve computation (adding, subtracting, multiplying, and dividing). Some problems are realistic; others are puzzles.

You will have the opportunity in most activities to try a problem-solving challenge. Watch for these icons.

This icon is a cue that the problem will be related to the topic of the activity.

This icon is a cue that the problem will provide a change of topic.

The Four-Stage Process

There are four stages that can be used to solve any problem: understanding the problem, developing a plan, trying the plan, and looking back.

Understanding the Problem

In this stage you should expect to feel puzzled. There are various reasons for feeling this way.

- You may not know the meanings of all the words.
- You may not understand the situation in the problem.
- You may be confused by unnecessary information.

Once you understand the problem, you should think about the problem and make an estimate of what the answer should be. This will help you arrive at a reasonable answer.

Developing a Plan

This is where you should decide on the plan of action that you are going to take to solve the problem. You may consider the following strategies:

- using objects
- using diagrams
- making a table
- working backwards
- using elimination
- using truth tables
- using an equation
- changing your point of view
- making an organized list
- using Venn diagrams
- simplifying a problem
- guessing, checking, and revising
- finding and applying a pattern
- acting out a problem

Note: The Appendix in Module 6 explains these strategies in detail. When you see a problem-solving icon in any module, you should turn to the Module 6 Appendix and review the problem-solving strategies.

Trying the Plan

In this stage you should try the plan and see if it works.

Be sure to work carefully and record your progress. You are encouraged to use a calculator to help with your calculations.

Note: While trying the plan, you should monitor your progress in order to determine if your plan will lead to a solution. You may find that the plan will not produce a solution, in which case a new plan will have to be developed.

Looking Back

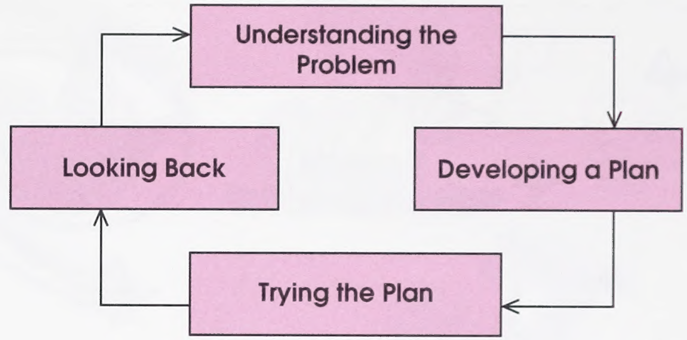
In this stage you should look back at the problem and compare your answer to the estimate you made in the first stage. Restate the problem using your answer.

Ask yourself these questions: "Did my plan work? Is my answer reasonable?"

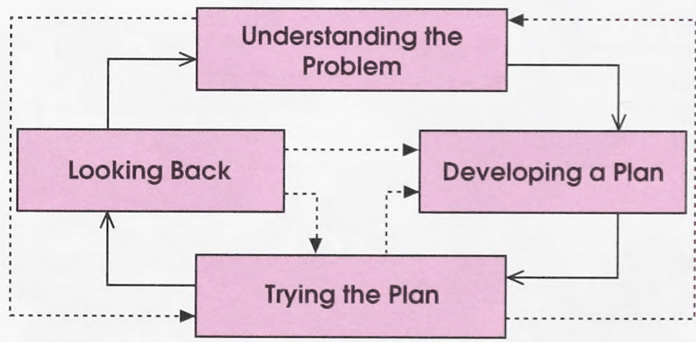
If you did not arrive at an answer, another strategy may work better. If your answer is unreasonable, you may have made errors while trying your plan.

Sequence of Stages

You usually approach a problem in the order outlined in the following diagram.



If you encounter difficulties in your original plan, or if you realize that another strategy will have better results, you may need to return to an earlier stage or use the stages in a different sequence.



CONTENTS

Module Overview

When you gaze on a beautiful flower garden or the greens at a golf course, you may not think of fertilizers. However, fertilizers are used extensively in gardening, landscaping, and agriculture. The three primary fertilizer nutrients are nitrogen, phosphorus, and potassium. During the last decade more than 1.43×18^8 t of these fertilizer nutrients were used in the world. Each container of chemical fertilizer is labelled with a three-term ratio that shows the percent of each of these nutrients. For example, a garden fertilizer may be labelled 6-12-12. This means that it contains 6% nitrogen, 12% phosphorus, and 12% potassium. A lawn fertilizer may be labelled 10-6-4. What does that mean?

Numbers are used to describe and measure many activities in this world, including gardening and landscaping. It is important that you become comfortable dealing with numbers and their applications.

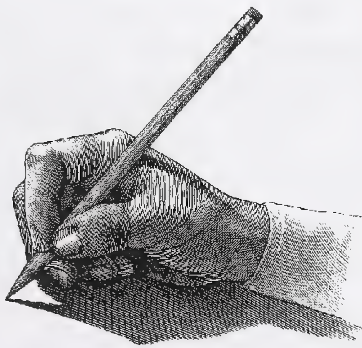
In this module you will work with proportional situations, powers, and roots. It will be a powerful module!

Evaluation

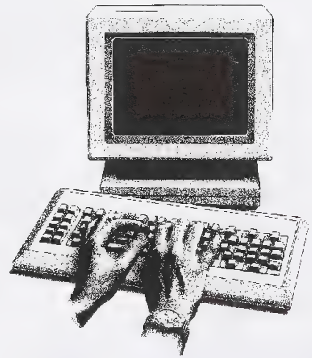
Your mark for this module will be determined by how well you complete the assignments at the end of each section and at the end of the module. In this module you must complete three assignments. The mark distribution is as follows:

Section 1 Assignment	**65 marks**
Section 2 Assignment	**25 marks**
Final Module Assignment	**10 marks**
TOTAL	**100 marks**

When doing the assignments, work slowly and carefully. You must do each assignment independently, but if you are having difficulties, you may review the appropriate section in this module booklet.



If you are working on a computer managed learning (CML) terminal, you will have a module test as well as a module assignment.



Note

There is a final supervised test at the end of this course. Your mark for the course will be determined by how well you do on the module assignments and the supervised final test.

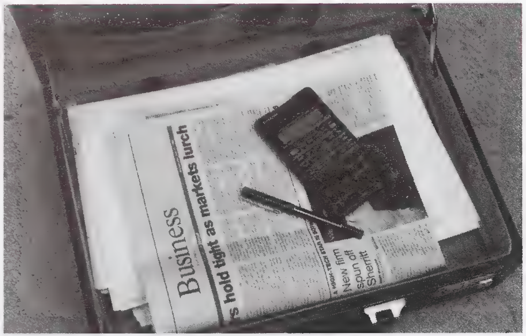
Section 1: Working with Proportional Situations

Have you ever helped shear a sheep? Have you ever knit a wool sweater? Some of the following facts about sheep and wool may surprise you. Sheep bred for their wool account for 50% of the world sheep population. A fine-wool sheep such as a merino can have 9000 wool follicles per square centimetre of skin. There are about 100 000 000 fibres per fleece. A merino can grow nearly 8 900 000 m of wool fibre per year!

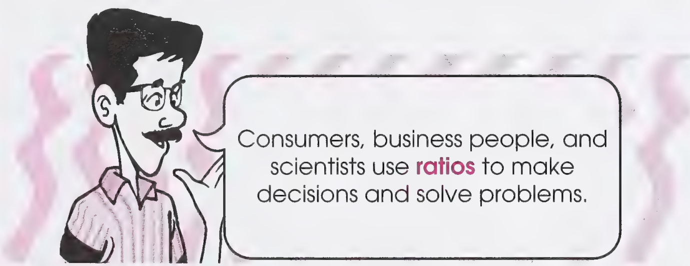
To appreciate these facts you must have a sense of ratios, rates, and percents.

In this section you will solve problems involving ratios, rates, and percents. You will use three-term ratios, unit rates, and fractional percents. You will investigate many situations in which ratios, rates, and percents are used—commissions, sales tax, tipping, simple interest, compound interest, and discounts. You will gain an appreciation for proportion and scale. You will explore scale models and drawings. You will make enlargements and reductions.

Activity 1: Ratios



Heidi owns shares in several companies. In order to monitor her investments, Heidi reads the business section of newspapers. She carefully notes data such as the price/earnings (P/E) ratios.



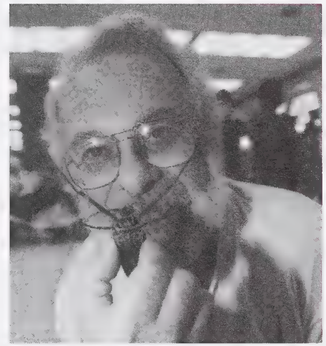
A ratio provides a comparison of quantities measured in the same unit.

The numbers in a ratio are called **terms**. The first number is the first term, the second number is the second term, and the third number is the third term.

A two-term ratio may be written in colon form or fraction form. A three-term ratio is written in colon form.

Generally, ratios are written in **lowest terms**. A ratio is in lowest terms if the terms are the lowest possible whole numbers.

Example 1


[1]

Beetles vary greatly in size. For example, a minute beetle is 0.02 cm long. A goliath beetle is 14.86 cm long.

What is the ratio of the length of the minute beetle to the length of the goliath beetle?

Solution

Method 1: Using the Colon Form of the Ratio

Step 1: Write the ratio of the length of the minute beetle to the length of the goliath beetle.

minute:goliath 0.02:14.86

[1] Steve Sanford, photo, *The Edmonton Journal*, 22 September 1996, B1. Reprinted by permission.

Step 2: Write each term as a whole number. Then write the ratio in lowest terms.

minute:goliath

$$
\begin{aligned}
&0.02 : 14.86 \\
&\downarrow \times 100 \quad \downarrow \times 100 \\
=\ &2 : 1486 \\
&\downarrow \div 2 \quad \downarrow \div 2 \\
=\ &1 : 743
\end{aligned}
$$

The ratio of the length of the minute beetle to the length of the goliath beetle is 1 to 743.

Method 2: Using the Fraction Form of the Ratio

Step 1: Write the ratio of the length of the minute beetle to the length of the goliath beetle.

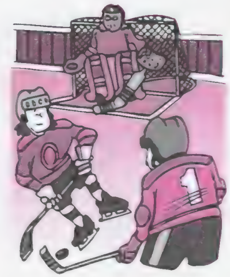
$\frac{\text{minute}}{\text{goliath}}$ $\frac{0.02}{14.86}$

Step 2: Write each term as a whole number. Then write the ratio in lowest terms.

$\frac{\text{minute}}{\text{goliath}}$ $\frac{0.02}{14.86} \overset{\times 100}{=} \frac{2}{1486} \overset{\div 2}{=} \frac{1}{743}$

The ratio of the length of the minute beetle to the length of the goliath beetle is 1 to 743.

Example 2



During the hockey season, a team had 33 wins, 24 losses, and 9 ties.

What is the ratio of wins to games played?

Solution

Method 1: Using the Colon Form of the Ratio

Step 1: Find the total number of games played during the hockey season.

$$33 + 24 + 9 = 66$$

The team played 66 games during the hockey season.

Step 2: Write the ratio of wins to games played.

wins:games played   33:66

Step 3: Write the ratio in lowest terms.

wins:games played

$$\begin{array}{ccc} 33 & : & 66 \\ \downarrow \div 33 & & \downarrow \div 33 \\ = 1 & : & 2 \end{array}$$

The ratio of the wins to games played is 1 to 2.

Method 2: Using the Fraction Form of the Ratio

Step 1: Find the total number of games played during the hockey season.

$$33+24+9=66$$

The team played 66 games during the hockey season.

Step 2: Write the ratio of wins to games played.

$\frac{\text{wins}}{\text{games played}}$ $\frac{33}{66}$

Step 3: Write the ratio in lowest terms.

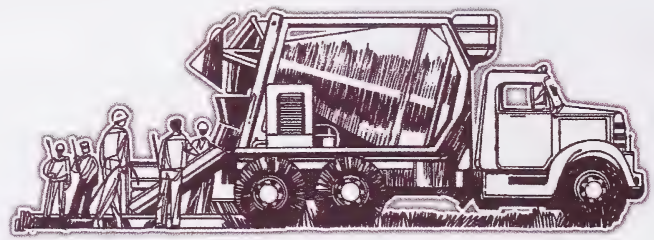
$\frac{\text{wins}}{\text{games played}}$

$$\frac{33}{66} \overset{\div 33}{\underset{\div 33}{=}} \frac{1}{2}$$

The ratio of wins to games played is 1 to 2.

Example 3

To make a concrete foundation, water is mixed with 300 kg of gravel, 450 kg of sand, and 100 kg of cement.



What is the ratio of gravel to sand to cement?

Solution

Step 1: Write the ratio of gravel to sand to cement. **Hint:** Be sure to put the numbers in the correct order.

gravel:sand:cement   300:450:100

Step 2: Write the ratio in lowest terms.

gravel:sand:cement

$$\begin{array}{cccccc} & 300 & : & 450 & : & 100 \\ & \downarrow \div 50 & & \downarrow \div 50 & & \downarrow \div 50 \\ = & 6 & : & 9 & : & 2 \end{array}$$

The ratio of gravel to sand to cement is 6 to 9 to 2.

1. Switzerland won 5 gold medals, 5 silver medals, and 5 bronze medals in the 1988 Winter Olympics held in Calgary.

   a. What is the ratio of gold medals to silver medals to bronze medals won by Switzerland in the 1988 Winter Olympics?

   b. What is the ratio of gold medals to the total medals won by Switzerland in the 1988 Winter Olympics?

2. Aerobic activity is important to maintain good health. In one day an adult inhales about 11.4 m^3 of air. About 0.57 m^3 of the oxygen in this air is absorbed into the bloodstream.

   What is the ratio of the oxygen absorbed to the air inhaled?

3. To produce a shade of brown, Mark mixed 2 L of red paint, 1.25 L of yellow paint, and 0.25 L of black paint.

   a. What is the ratio of red to yellow to black paint?

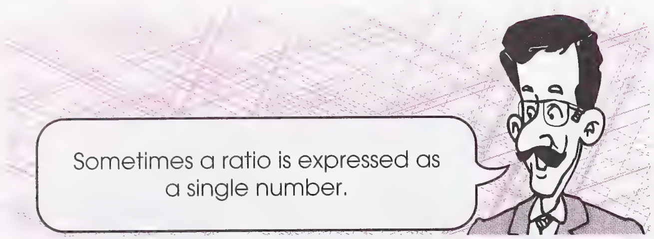
   b. What is the ratio of red to yellow to black to brown paint?

4. The largest of all known diamonds is the Cullinan. The Cullinan had a mass of 3106 carats before it was cut into 106 stones. The largest of the stones produced from the Cullinan is the Great Star of Africa, which has a mass of 530.2 carats. The Great Star of Africa was set in the British royal sceptre.

   What is the ratio of the mass of the Great Star of Africa to the mass of the Cullinan?

Check your answers by turning to the Appendix.

Use the Internet to discover more about the Cullinan, the Great Star of Africa, and other famous diamonds.



Example 4

Regardless of the size of a circle, the ratio of the circumference to the diameter is always the same. The Greek letter π (pi) is used to represent the first term of this ratio. The second term is understood to be 1.

Using a tailor's measuring tape, Wade measured the circumference and diameter of a round table. The circumference was 4.71 m and the diameter was 1.5 m.

Using Wade's measurements, calculate π, the ratio of the circumference to the diameter of the table. Express the ratio as a single number.

Solution

Step 1: Write the ratio of the circumference to the diameter.

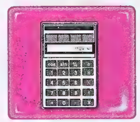
$$\frac{\text{circumference}}{\text{diameter}} \qquad \frac{4.71}{1.5}$$

Step 2: Use a calculator to express the ratio as a single number.

4 . 7 1 ÷ 1 . 5 =

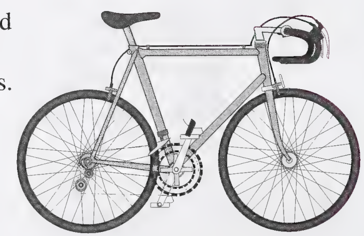
3.14

Using Wade's measurements, π is about 3.14.

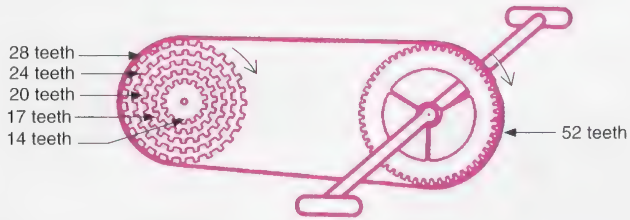


Use a calculator to answer questions 5 and 6.

5. On a five-speed or a ten-speed bicycle, a chain connects the front gears and the back gears.



This is a diagram of the gears on a five-speed bicycle.



Notice that there is one gear on the pedal and five gears on the back wheel. The gears can be connected in different positions.

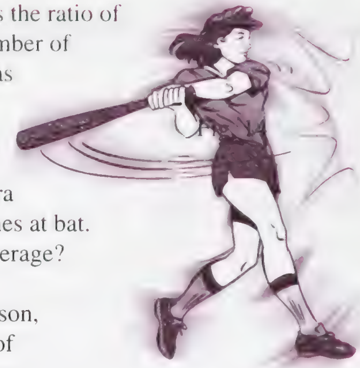
The **gear ratio** is the ratio of the number of teeth on the front gear to the number of teeth on the back gear. It is expressed as a single number. The second term is understood to be 1.

a. Complete a chart like this. **Hint:** Express the gear ratio as a decimal number rounded to three decimal places.

Position of Gear	Number of Teeth on Front Gear	Number of Teeth on Back Gear	Gear Ratio
1st	52	28	
2nd	52	24	
3rd	52	20	
4th	52	17	
5th	52	14	

b. Compare the gear ratios in the chart. Which gear has the greatest gear ratio? Which gear has the lowest gear ratio?

6. A baseball batting average is the ratio of the number of hits to the number of times at bat. It is expressed as a decimal number rounded to three decimal places.



a. In a baseball season, Tara had 22 hits out of 75 times at bat. What was her batting average?

b. In the same baseball season, Jocelyn had 15 hits out of 60 times at bat. What was her batting average?

c. Who is the better hitter, Tara or Jocelyn?

Check your answers by turning to the Appendix.

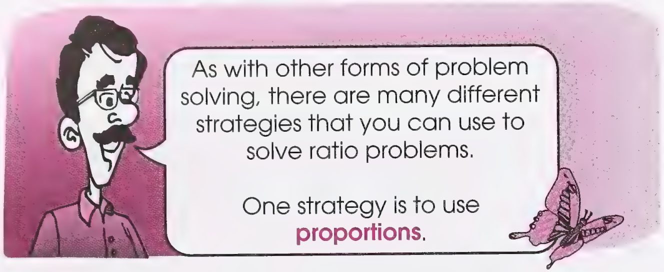
Use the Internet to find the batting average of major league baseball players. One site you may find useful is the American League site at the following uniform resource locator (URL):

http://www.majorleaguebaseball.com/al

Following is the URL for the National League:

http://www.majorleaguebaseball.com/nl

Using Proportional Reasoning



A proportion is an equation that shows two ratios are equivalent.

You can use symbols such as $\square$ and $\bigcirc$ to represent the missing terms in a proportion.

Example 1

Theoretically, the ratio of the height a person can jump on the Moon to the height a person can jump on Earth is 6 to 1.

If a pole vaulter can jump 5.5 m on Earth, how high could she jump on the Moon? **Note:** Assume that breathing is not a factor.

Solution

Method 1: Using the Colon Form

Step 1: Write a proportion. Let the height on the Moon be $\square$.

height on the Moon:height on Earth

$$6:1 = \square:5.5$$

Step 2: Find the missing value.

height on the Moon:height on Earth

$$\begin{array}{ccc} 6 & : & 1 \\ \downarrow \times 5.5 & & \downarrow \times 5.5 \\ =\boxed{33} & : & 5.5 \end{array}$$

The pole vaulter could jump 33 m on the Moon.

Method 2: Using the Fraction Form

Step 1: Write a proportion. Let the height on the Moon be $\square$.

$\dfrac{\text{height on the Moon}}{\text{height on Earth}}$

$$\frac{6}{1} = \frac{\square}{5.5}$$

Step 2: Find the missing value. Use this reasoning: Because 1 was multiplied by 5.5, multiply the other term by 5.5.

$$\frac{\text{height on the Moon}}{\text{height on Earth}} \qquad \frac{6}{1} \underset{\times 5.5}{\overset{\times 5.5}{=}} \frac{\boxed{33}}{5.5}$$

The pole vaulter could jump 33 m on the Moon.

Example 2

If stereo speakers are to have good acoustics, the ratio of their depth to their width to their height is 1 to 2 to 3.

If a speaker is 90 cm high, how deep and wide should it be to have good acoustics?

Solution

Step 1: Write a proportion. Let $\boxed{}$ be the depth and $\bigcirc$ be the width.

depth:width:height

$$1 : 2 : 3$$
$$= \boxed{} : \bigcirc : 90$$

Step 2: Find the missing values. Use this reasoning: Because 3 was multiplied by 30 to get 90, multiply the other terms by 30.

depth:width:height

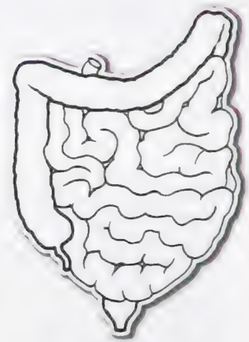
$$\begin{array}{ccccc} 1 & : & 2 & : & 3 \\ \downarrow{\times 30} & & \downarrow{\times 30} & & \downarrow{\times 30} \\ = \boxed{30} & : & \boxed{60} & : & 90 \end{array}$$

This speaker should be 30 cm deep and 60 cm wide.

Example 3

The ratio of the length of an adult's small intestine to the length of the person's large intestine is about 4 to 1.

If a person has a total of 7.5 m in small and large intestines, how long is the small intestine? How long is the large intestine?



Solution

Step 1: Write the ratio of the length of the small intestine to the length of the large intestine to the total length of the intestines. **Note:** $4 + 1 = 5$

small:large:total $\qquad$ 4:1:5

Step 2: Write a proportion. Let the length of the small intestine be ___ and the length of the large intestine be ___.

small:large:total

$$4 : 1 : 5$$
$$= \_\_ : \_\_ : 7.5$$

Step 3: Find the missing values. Use this reasoning: Because 5 was multiplied by 2.5 to get 7.5, multiply the other terms by 2.5.

small:large:total

$$\begin{array}{ccccc} 4 & : & 1 & : & 5 \\ \downarrow \times 1.5 & & \downarrow \times 1.5 & & \downarrow \times 1.5 \\ = 6 & : & 1.5 & : & 7.5 \end{array}$$

The length of the small intestine is 6 m. The length of the large intestine is 1.5 m.

7. A bag contains red marbles, white marbles, and blue marbles. The ratio of the red marbles to white marbles to blue marbles is 2 to 3 to 4.

If there are 12 blue marbles in the bag, how many red marbles are there? How many white marbles are there?

8. The amount of gold in jewellery is measured in karats (K). This measure is a ratio expressed as a single number. The second term is understood to be 24. For example, the mark of 10 K means that the ratio of the mass of the gold in the jewellery to the total mass of the metals is 10 to 24.

If a ring is marked 14 K and the ring (without any stones) has a mass of 72 g, what is the mass of gold in the ring?

9. Sterling silver is an alloy of silver and copper in the ratio of 37 to 3.

If a sterling silver goblet has a mass of 800 g, how much silver is in the goblet? How much copper is in the goblet?

10. In the Appendix, find and read the article “Magic Clay Turned Bulb into Bauble.” Then answer the following questions.

a. If the piece is one-fifth gold and four-fifths silver, what is the ratio of gold to silver?

b. If $3\frac{3}{4}$ ounces is equivalent to 110 g, what is the mass of the gold in the bulb?

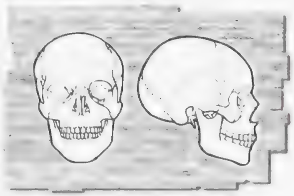
11. A one-cent coin minted before 1860 contains copper, tin, and zinc in the ratio of 95:4:1 and has a mass of 4.5 g.

a. What is the mass of copper in the coin?
b. What is the mass of tin in the coin?
c. What is the mass of zinc in the coin?

12. Meredith and Raschid invest in a clothing store in the ratio of 2 to 3. They decide to divide the profits in the same ratio.

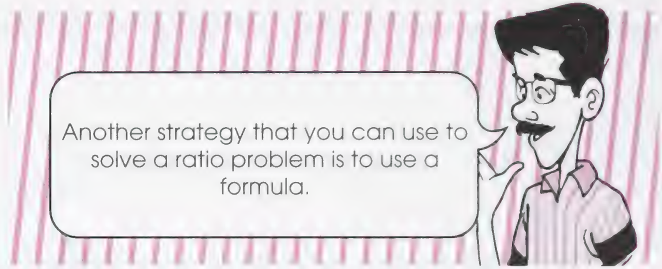
If the store made $1500 in profits, how much did each partner receive?

13. An adult's body contains 206 bones. The ratio of the number of bones in the person's head to the number of bones in the person's body is about 1 to 7.



About how many bones are there in an adult's head?

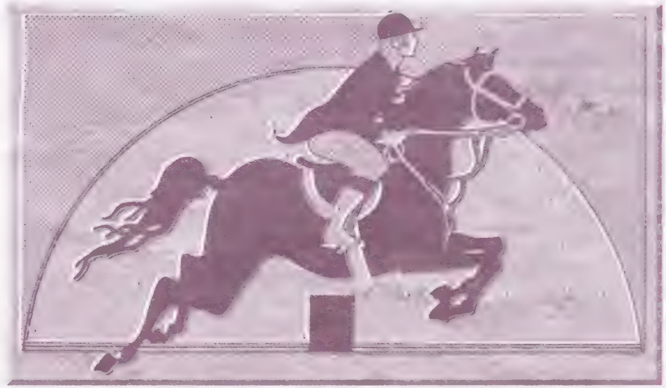
Check your answers by turning to the Appendix.



Example 4

Theoretically, the ratio of the height a horse could jump on the Moon to the height a horse can jump on Earth is 6 to 1.

If a horse can jump 3.5 m on Earth, how high could the horse jump on the Moon? **Note:** Assume that breathing is not a factor.



Solution

Step 1: Write the ratio of the height a horse could theoretically jump on the Moon to the height a horse can jump on Earth as a single number. **Hint:** Express the ratio as a whole number.

$$\frac{\text{Moon}}{\text{Earth}} \qquad \frac{6}{1} = 6$$

Step 2: Write a formula that shows how the height jumped on the Moon is related to the height jumped on Earth.

Let m be the height (in m) jumped on the Moon.
Let e be the height (in m) jumped on Earth.

$$m = 6e$$

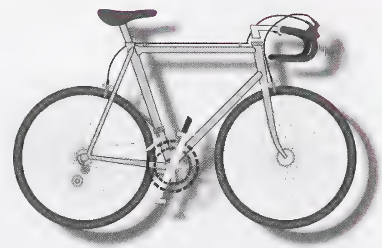
Step 3: Calculate the height a horse could theoretically jump on the Moon if it could jump 3.5 m on Earth.

$$\begin{aligned} m &= 6e \\ &= 6(3.5) \\ &= 21 \end{aligned}$$

Theoretically, the horse could jump 21 m on the Moon.

Example 5

When a ten-speed bicycle is in first gear, 7 revolutions of the pedal make 10 revolutions of the wheel.



How many pedal revolutions are needed for 120 wheel revolutions?

Solution

Step 1: Express the ratio of pedal revolutions to wheel revolutions as a single number. **Hint:** Express the ratio as a fraction or decimal number.

$$\frac{\text{pedal revolutions}}{\text{wheel revolutions}} \qquad \frac{7}{10} \quad \textbf{or} \quad \frac{7}{10} = 0.7$$

Step 2: Write a formula that shows how the number of pedal revolutions is related to the number of wheel revolutions.

Let p be the number of pedal revolutions.
Let w be the number of wheel revolutions.

$$p = \frac{7}{10}w \quad \textbf{or} \quad p = 0.7w$$

Step 3: Calculate the number of pedal revolutions for 120 wheel revolutions.

$$p = \frac{7}{10}w \qquad \textbf{or} \qquad p = 0.7w$$

$$= \frac{7}{10}(120) \qquad\qquad = 0.7(120)$$

$$= 84 \qquad\qquad = 84$$

The number of pedal revolutions needed is 84.

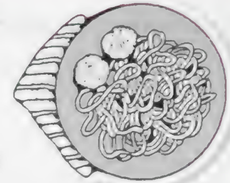
14. When you make orange juice from cans of frozen concentrate, you add 3 cans of water for every can of concentrate.

How many cans of water would you add if you had 2 cans of concentrate?

15. When you make an oil and vinegar salad dressing, you add 1 part of white vinegar for every 2 parts of vegetable oil.

If you use 60 mL of vegetable oil for the dressing, how much white vinegar should you use?

16. Pasta expands when it cooks. Therefore, you use 1 mL of uncooked pasta for every 3 mL of cooked pasta.



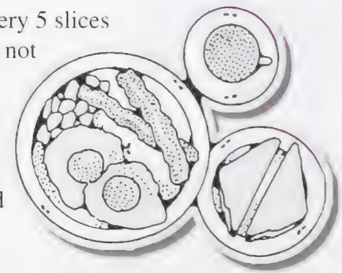
How much uncooked pasta would you use in order to make 900 mL of cooked pasta?

Check your answers by turning to the Appendix.

Now Try This

Use a problem-solving strategy to answer the following question.

17. A restaurant burns 1 out of every 5 slices of toast. The burned slices are not used.



If each person must receive 2 slices of toast, how many slices of bread must be toasted to serve 90 people?

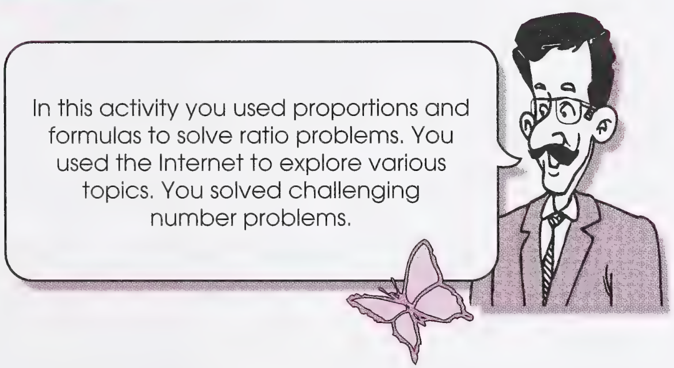
Check your answer by turning to the Appendix.

Use a problem-solving strategy to answer the following question.

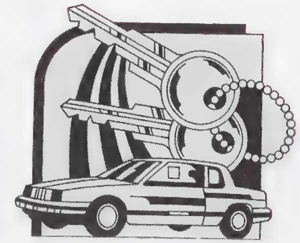
18. Five friends have been borrowing and lending money to each other. Jane owes Gilbert $1. Tanya owes Katherine $1. Katherine owes Gilbert $2. Gilbert owes Lloyd $3. Tanya owes Lloyd $5. Lloyd owes Katherine $1. Tanya owes Gilbert $2. Katherine owes Tanya $1. Lloyd owes Jane $3.

If each friend receives a $10 allowance and pays off all his or her debts, who will have the most money? Who will have the least money?

Check your answer by turning to the Appendix.

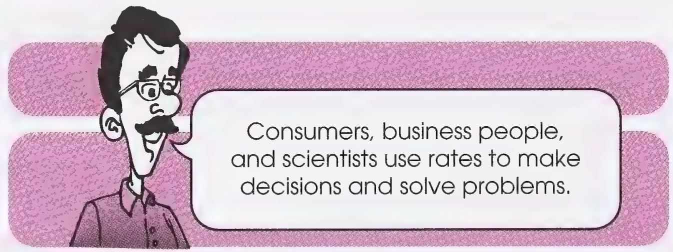


Activity 2: Rates



Lorne wants to purchase a new car. He decides to do some research to discover which car best suits his needs. He reads consumer reports and manufacturer's brochures. He compares data such as gas consumption **rates**.

A rate provides a comparison of quantities measured in different units.



Usually rates are written as **unit rates**. Unit rates have a second term of 1, but the 1 is not written.

Example 1

The following advertisements appeared in a grocery store flyer.

Which is the better buy—the 10-kg bag or the 4-kg bag?

Solution

Step 1: Write the unit cost of the 10-kg bag.

$$\frac{\text{cost (\$)}}{\text{mass (kg)}} \qquad \frac{6.99}{10} \overset{\div 10}{\doteq} \frac{0.70}{1}$$

The unit cost is about $0.70/kg.

Step 2: Write the unit cost of the 4-kg bag.

$$\frac{\text{cost (\$)}}{\text{mass (kg)}} \qquad \frac{2.49}{4} \overset{\div 4}{\doteq} \frac{0.62}{1}$$

The unit cost is about $0.62/kg.

Step 3: Compare the unit costs.

$$0.62 < 0.70$$

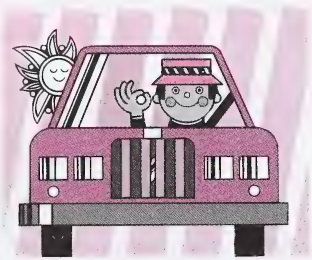
The 4-kg bag is the better buy.

1. Marlene and Jeannie each have after-school jobs. Last week Marlene worked for 8 h and earned $44. Jeannie worked for 6 h and earned $39.

   Which girl earned the better rate of pay?

2. Arvind travelled 740 km in 8 h, after rest stops were subtracted. Tim travelled 650 km in 7 h, after rest stops were subtracted.

   Who travelled at the faster rate of speed?



3. Density is the mass per volume of a substance. Density is usually measured in g/cm^3.

   a. If 40 cm^3 of water has a mass of 40 g, what is the density of water?

   b. If 50 cm^3 of cast iron has a mass of 350 g, what is the density of cast iron?

   c. If 60 cm^3 of oak has a mass of 40 g, what is the density of oak?

   d. Which substance is densest—water, cast iron, or oak?

   e. How does the density of a substance affect its ability to float on water?

Check your answers by turning to the Appendix.

4. In the 1996 Summer Olympics in Atlanta, Canada's Donovan Bailey won the 100-m race in 9.84 s. America's Michael Johnson won the 200-m race in 19.32 s.

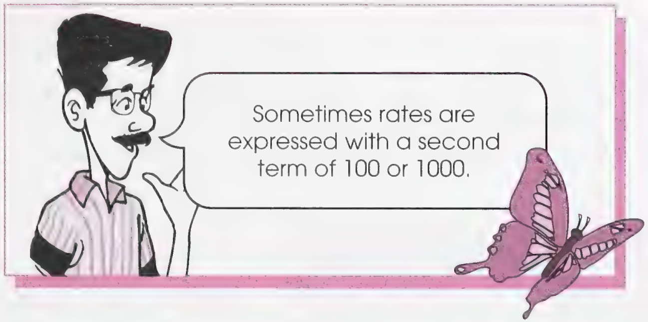
NICK DIDLICK VANCOUVER SUN

   a. Using proportional reasoning, which man appeared to be the faster runner in 1996?

b. Read the article entitled "Is Bailey the Fastest? Of Course He Is" in the Appendix of this module.

Explain in your own words why many people feel Donovan Bailey is a faster runner than Michael Johnson.

Check your answer by turning to the Appendix.



Example 2

Gas consumption is usually measured in L/100 km.

If Gaston's car consumed 68 L of fuel on a 500-km trip, what was the car's rate of fuel consumption?

Solution

Step 1: Write the rate of gas consumption.

consumption (L):distance (km)   68:500

Step 2: Write the rate with a second term of 100.

consumption (L):distance (km)

$$68 : 500 \xrightarrow{\div 5,\ \div 5} = 13.6 : 100$$

The gas consumption rate was 13.6 L/100 km.

Use a calculator to answer questions 5 and 6.

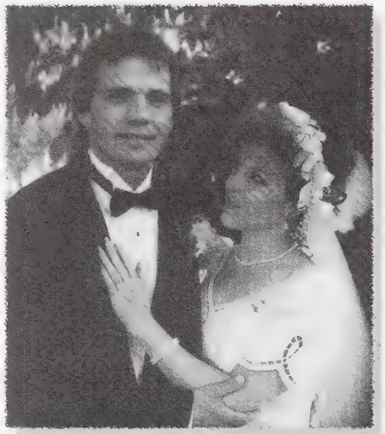
5. The cost of deli meats is usually expressed as dollars per 100 g.

   a. Reba bought 300 g of salami for \$4.47. What was the cost of the salami?

b. Matt bought 250 g of black forest ham for \$3.75. What was the cost of the black forest ham?

6. Birth rates, death rates, and marriage rates are expressed as frequencies per 1000 population.

a. There were 184 096 marriages in Canada in 1985. If the population in Canada in 1985 was about 25 942 000, what was the marriage rate?



DON KLEIN

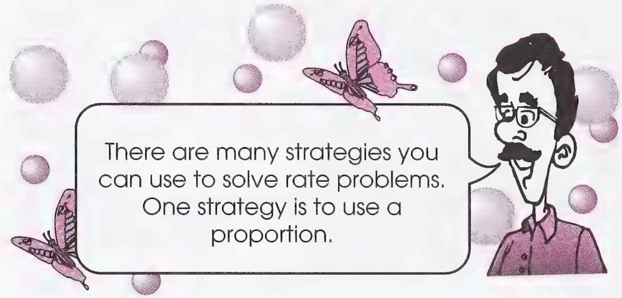
b. There were 160 616 marriages in Canada in 1995. If the population in Canada in 1995 was about 29 606 000, what was the marriage rate?

Check your answers by turning to the Appendix.

Use the search button on Statistics Canada's web site to discover the birth rate, death rate, and marriage rate in Canada for various years. Following is the uniform resource locator of Statistics Canada's web site.

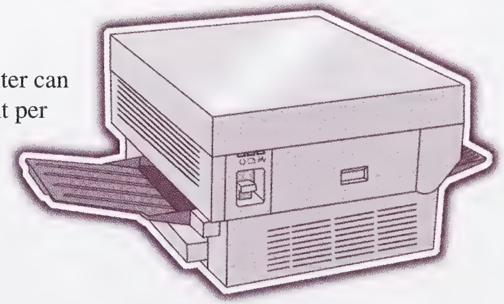
http://www.statcan.ca/start.html

Using Proportional Reasoning



Example 1

A certain laser printer can print 4 pages of text per minute. At this rate, how long will it take to print a 60-page report?



Solution

Method 1: Using the Colon Form

Step 1: Write a proportion.

pages:time (min)

$$4 : 1$$
$$= 60 : \square$$

Step 2: Find the missing term.

pages:time (min)

$$\begin{array}{ccc} 4 & : & 1 \\ \downarrow \times 15 & & \downarrow \times 15 \\ = 60 & : & \boxed{15} \end{array}$$

At this rate, it will take 15 min to print the report.

Method 2: Using the Fraction Form

Step 1: Write a proportion.

$$\frac{\text{pages}}{\text{time (min)}} \qquad \frac{4}{1} = \frac{60}{\boxed{}}$$

Step 2: Find the missing term.

$$\frac{\text{pages}}{\text{time (min)}} \qquad \frac{4}{1} \overset{\times 15}{=} \frac{60}{\boxed{15}} \ (\times 15)$$

At this rate, it will take 15 min to print the report.

Example 2

The fuel consumption rate for Jackson's vehicle is 17 L/100 km. At this rate, how much gasoline will be used in a 250-km trip?

Solution

Method 1: Using the Colon Form

Step 1: Write a proportion.

gas consumed (L):distance travelled (km)

$$\begin{array}{l} 17:100 \\ = \boxed{}:250 \end{array}$$

Step 2: Find the missing value.

gas consumed (L):distance travelled (km)

$$\begin{array}{ccc} 17 & : & 100 \\ \downarrow \times 2.5 & & \downarrow \times 2.5 \\ = 42.5 & : & 250 \end{array}$$

Jackson's car will use 42.5 L of gas.

Method 2: Using the Fraction Form

Step 1: Write a proportion.

$$\frac{\text{gas consumed (L)}}{\text{distance travelled (km)}} \qquad \frac{17}{100} = \frac{\square}{250}$$

Step 2: Find the missing value.

$$\frac{\text{gas consumed (L)}}{\text{distance travelled (km)}} \qquad \frac{17}{100} \underset{\times 2.5}{\overset{\times 2.5}{=}} \frac{42.5}{250}$$

Jackson's car will use 42.5 L of gas.

7. Samuel can enter data on a computer at the rate of 125 words per minute.

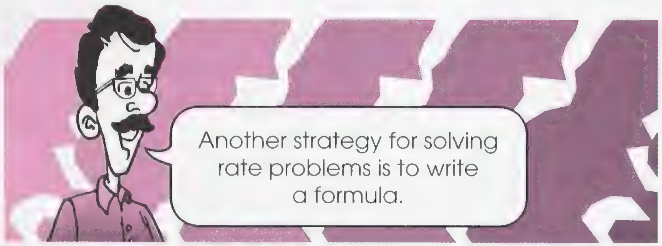
At this rate, how long will it take Samuel to enter a 2000-word report?

8. Smoked turkey costs \$2.25/100 g at Olson's Deli.

a. At this price, how much would 400 g of smoked turkey cost?

b. How much smoked turkey meat could you buy for \$20?

Check your answers by turning to the Appendix.



Example 3

If steak costs \$8.60/kg, how much does 5 kg cost?

Solution

Step 1: Write a formula. Let c be the cost (in dollars) and m be the mass (in kg).

$$c = 8.60m$$

Step 2: Find the cost of 5 kg of steak.

$$\begin{aligned} c &= 8.60m \\ &= 8.60(5) \\ &= 43.00 \end{aligned}$$

The cost is \$43.00.

Use a calculator to answer questions 9 to 12.

9. The following table lists the cruising speed of three types of planes.

Type of Plane	Cruising Speed (km/h)
DC10	880
B747	920
B737	840

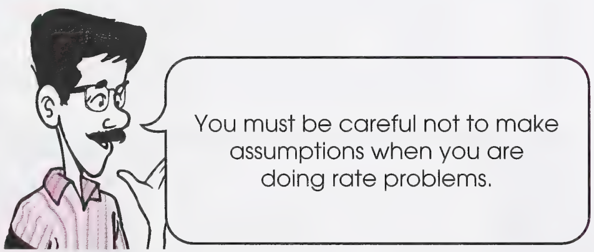
Determine how far each plane will travel in 3 h. **Hint:** Use the formula $d = st$, where d is the distance travelled (in km), s is the speed (in km/h), and t is the time (in h).

10. The very slow giant tortoise of Mauritius travels at a speed of about 0.005 km per minute. How far will the tortoise travel in 3 h? **Hint:** Use the formula $d = st$.

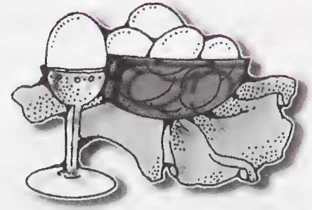
11. The human heart beats about 72 times per minute. At this rate, how many times does the heart beat in 30 s?

12. Your eye blinks about 25 times each minute. About how many times does your eye blink in an hour?

Check your answers by turning to the Appendix.



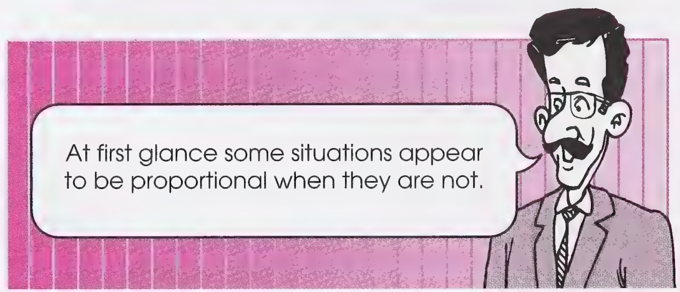
13. It takes 3 min to boil one egg. How long will it take to boil 6 eggs?



14. Read the Pickles cartoon which is reprinted in the Appendix of this module.

Explain what error in reasoning Sylvia made.

Check your answer by turning to the Appendix.



You can use a graph to decide whether or not a situation is proportional. The graph of a proportional situation is a straight line passing through the origin, $(0, 0)$, and leaning upwards and to the right.

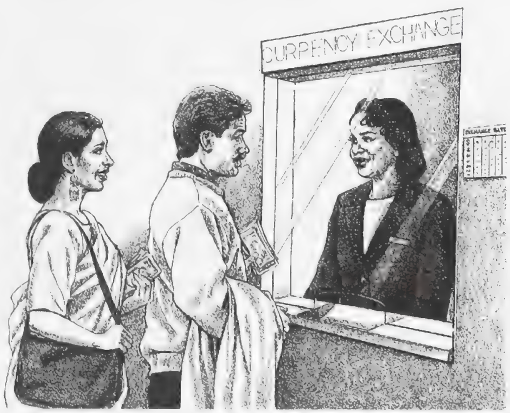
Use a spreadsheet program such as *ClarisWorks*™ to answer questions 15 to 18. **Note:** In Module 2, you discovered how to make a graph using a spreadsheet.

If you do not have access to a computer and a spreadsheet program, do the calculations and graphs by hand.

15. Roy and Krishnie made a trip to the United States. Before they went they exchanged some of their Canadian money for American money.

That day, C$1 was worth U.S.$0.75.

Note: C$1 is read as, "one dollar in Canadian currency."



One dollar in Canadian currency may be written as C$1, CAN$1, Can$1, $1 (Can), or CDN$1.

a. Complete a table like the following.

Amount in Canadian Dollars	Amount in American Dollars
0.00	
1.00	
2.00	
3.00	
4.00	

b. Make a graph to describe the situation.

c. Is this a proportional situation? Explain.

16. Judith has an after-school job working at a dry cleaners. When she works 1 h, she earns $5.50.

a. Complete a table like the following to describe this situation.

Number of Hours Worked	Earnings ($)
0	
2	
4	
6	
8	

b. Make a graph to describe the situation.

c. Is this situation a proportional situation? Why or why not?

17. Susan and Julie are sisters. When Susan was born, Julie was 4 years old.

a. Complete a table like the following.

Julie's Age	Susan's Age
0	
2	
4	
6	
8	

b. Make a graph to describe the situation.

c. Is this situation a proportional situation? Why or why not?

18. Ted and Stephen were running equally fast around a circuit. Ted started first; therefore, when Ted had run 3 laps, Stephen had run 1 lap.

a. Complete a table like the following to describe this situation.

Number of Laps Ted Ran	Number of Laps Stephen Ran
0	
1	
2	
3	
4	
5	

b. Make a graph to describe the situation.

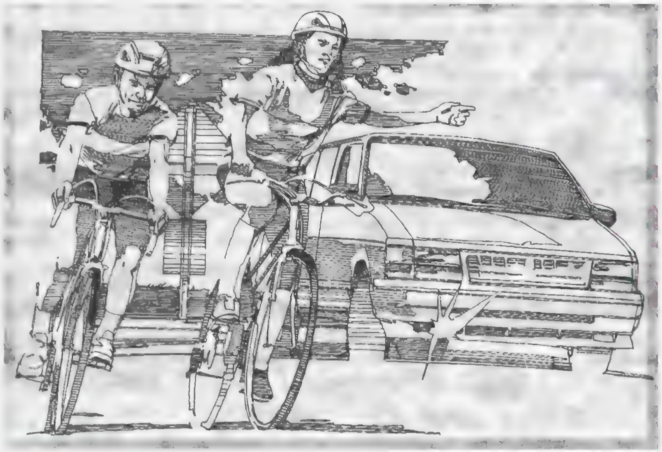
c. Is this situation a proportional situation? Why or why not?

Check your answers by turning to the Appendix.

Now Try This

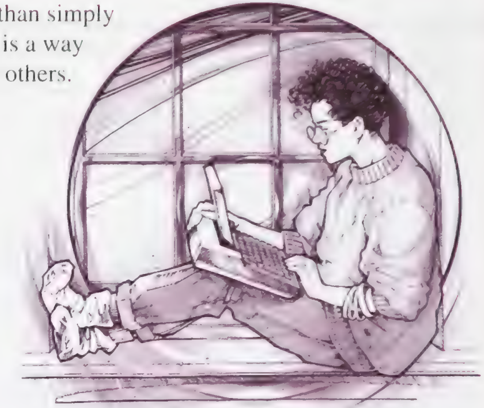
Use a problem-solving strategy to answer the following question.

19. Jason and Lisa travelled from their house to the lake. Jason left the house at 08:00 and jogged at an average speed of 6 km/h. Lisa left the house an hour later and biked at an average speed of 18 km/h. They reached the lake at the same time. How far was the lake from the house?



Check your answer by turning to the Appendix.

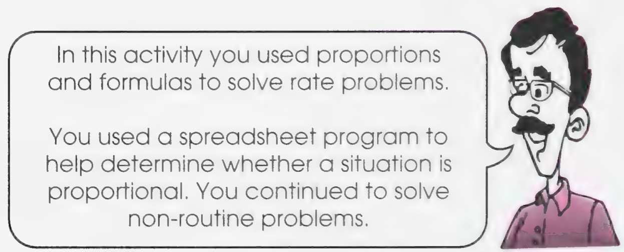
The Internet is more than simply a tool for research; it is a way to communicate with others.



You may wish to find out more about what other students are doing in math or try some challenging problems. The following sites may be helpful:

http://www.schoolnet.ca/math_sci/math/

http://forum.swarthmore.edu/students/



Activity 3: Percents

Many people today are concerned about what they eat. For this reason, manufacturers label many of their products with nutritional information.

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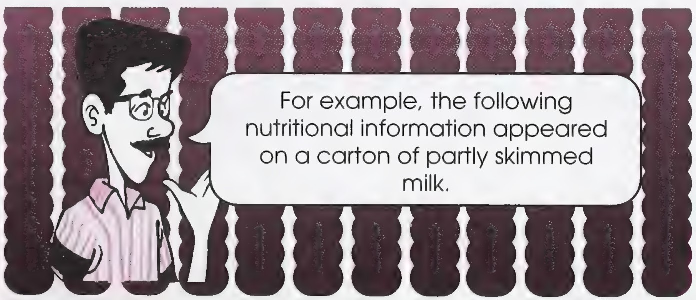
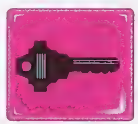


NUTRITION INFORMATION

Per 250-mL Serving	
Energy	108 Cal/450 kJ
Protein	8.5 g
Fat	2.7 g
Polyunsaturates	0.1 g
Monounsaturates	0.8 g
Saturates	1.7 g
Cholesterol	11 mg
Carbohydrates	12 g

Percentage of Recommended Daily Intake	
Vitamin A	11%
Vitamin D	44%
Thiamin	8%
Riboflavin	25%
Niacin	10%
Vitamin B_6	6%
Folacin	6%
Vitamin B_{12}	45%
Pantothenate	11%
Calcium	29%
Phosphorous	23%
Magnesium	14%
Zinc	11%

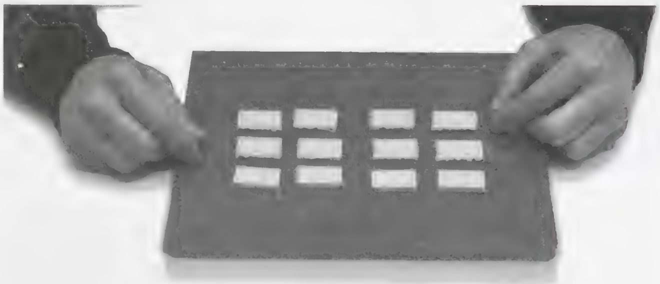
Notice that some of the information is provided as **percents**.



A percent is a special ratio. It has 100 as the second term, but "100" is not written. Instead, a percent symbol is used.

View the video program *The Percent* from the series *Mathways*. It provides a good overview of percent.

1. A certain type of chocolate bar can easily be divided into 12 equal squares.



   a. If Ron is given 6 squares of this kind of chocolate bar, what percent of the chocolate bar does he have?

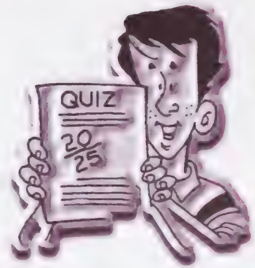
   b. If Ron is given 4 squares of this kind of chocolate bar, what percent of the chocolate bar does he have?

   c. If Ron is given 15 squares of this kind of chocolate bar, what percent of the chocolate bar does he have?

Check your answers by turning to the Appendix.

Example 1

Patrick scored 20 out of 25 on one quiz and 25 out of 30 on another quiz. On which quiz did Patrick do better?



Solution

Step 1: Express each ratio as a decimal number. **Hint:** You may use a calculator.

Quiz 1

$$\frac{20}{25} = 0.8$$

Quiz 2

$$\frac{25}{30} \doteq 0.83$$

Step 2: Express each ratio as an equivalent ratio with a second term of 100.

Quiz 1

$$\frac{20}{25} \doteq 0.8 = \frac{80}{100}$$

Quiz 2

$$\frac{25}{30} \doteq 0.83 \doteq \frac{83}{100}$$

Step 3: Express each ratio as a percent.

Quiz 1

$$\frac{20}{25} = 0.8 = \frac{80}{100} = 80\%$$

Quiz 2

$$\frac{25}{30} \doteq 0.83 \doteq \frac{83}{100} \doteq 83\%$$

Step 4: Compare the percents.

$83\% > 80\%$

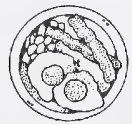
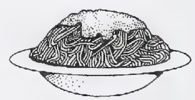
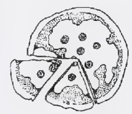
Patrick did better on the second quiz.

Use a calculator to answer question 2.

2. Salt and other sodium compounds are used in processed foods for preserving, flavouring, and stabilizing other ingredients. Sodium is also present naturally in some foods.

Because excessive amounts of sodium may contribute to high blood pressure and strokes in some people, the recommended maximum daily amount of sodium is about 2400 mg (about 5 mL of table salt).

a. Complete a table like the following.

Serving	Amount of Sodium per Serving* (mg)	Percentage of Recommended Maximum Daily Amount (%)
 cereal and milk	225	
 bacon, eggs, and hashbrowns	1000	
 pasta and sauce	400	
 personal pizza	1100	
 burger and fries	1000	

*__Note:__ The amount of sodium per serving varies with the ingredients.

b. Vern ate bacon, eggs, and hashbrowns for breakfast. He had a personal pizza for lunch. For supper he had a burger and fries. Calculate the percentage of the maximum daily recommended amount of sodium he consumed.

c. Deepa ate cereal and milk for breakfast. For lunch he had a personal pizza. For supper he had pasta and sauce. Calculate the percentage of the daily recommended amount of sodium he consumed.

d. Who consumed less sodium that day—Vern or Deepa?

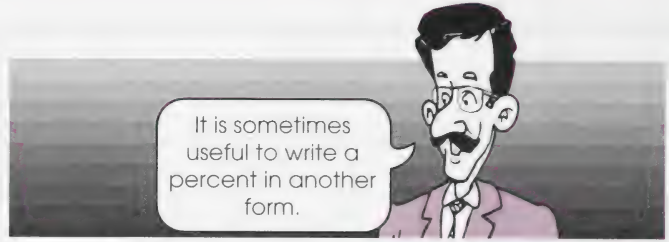
e. Did either Vern or Deepa consume more than the recommended daily maximum of sodium? If so, who?

Check your answers by turning to the Appendix.

You may use the Internet to find the amounts of sodium and other food nutrients in various foods. One site you may find interesting is the Nutrient Data Laboratory of the Agricultural Research Service:

http://www.nal.usda.gov/fnic/foodcomp/

Choose "Look Up" from the site. Enter the name of your food on the search page. Narrow your choice from the given items. You will be asked to choose the size of your sample; then select "Report." A detailed list of the food's nutrients will be given.



Example 2

The average life span of a domestic sheep is 150% of the life span of a domestic goat. Express this percent as a decimal number.

Solution

$$\begin{aligned} 150\% &= 150 \div 100 \\ &= 1.5 \end{aligned}$$

The average life span of a domestic sheep is 1.5 times the life span of a domestic goat.

Example 3

The size of Mercury is 6.25% of the size of Earth. Express this percent as a decimal number.

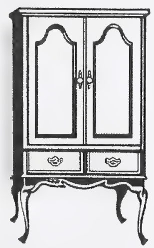
Solution

$$\begin{aligned} 6.25\% &= 6.25 \div 100 \\ &= 0.0625 \end{aligned}$$

The size of Mercury is 0.0625 times the size of Earth.

Example 4

The Smiths sold an antique chest for $133\frac{1}{3}\%$ of their purchase price. Express this percent as a mixed number.



Solution

$$\begin{aligned} 133\frac{1}{3}\% &= 133\frac{1}{3} \div 100 \\ &= \frac{400}{3} \div 100 \\ &= \frac{400}{3} \times \frac{1}{100} \\ &= 1\frac{1}{3} \end{aligned}$$

$$\frac{\overset{4}{\cancel{400}} \times 1}{3 \times \underset{1}{\cancel{100}}} = \frac{4}{3} = 1\frac{1}{3}$$

The Smiths sold the antique chest for $1\frac{1}{3}$ times their purchase price.

3. **a.** The area of Lake Superior is 400% of the area of Lake Ontario. Express this percent as a whole number.

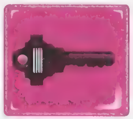
   b. The mass of 1 L of milk is 156% of the mass of 1 L of gasoline. Express this percent as a decimal number.

   c. Raymond won 87.5% of the races in which he competed. Express this percent as a decimal number.

   d. The flying speed of the spine-tail swift is $16\frac{2}{3}\%$ of the flying speed of some jet planes. Express this percent as a fraction.

Check your answers by turning to the Appendix.

Commission



Commission is the pay an employee earns based on a percentage of the sales made by that employee.

Example 1

Dan earns a commission of \$7200 on a house that he sells. If he sells the house for \$120 000, what rate of commission does he earn?

Solution

Method 1: Using a Proportion

Step 1: Write a proportion.

commission:selling price

$$7200:120\,000$$
$$= \boxed{} :100$$

Step 2: Find the missing value.

commission:selling price

$$7200 : 120\,000$$
$$\div 1200 \quad \div 1200$$
$$= \boxed{6} : 100$$

Step 3: Write the percent.

$$6:100 = 6\%$$

Dan earns a commission of 6%.

Method 2: Using a Formula

Step 1: Write a formula.

Let c be the commission (in dollars).
Let r be the rate of commission (as a decimal number).
Let s be the selling price (in dollars).

$$c = rs$$

Step 2: Substitute the given values into the formula.

$$c = rs$$
$$7200 = r(120\,000)$$

Step 3: To solve for r, divide each side of the equation by 120 000.

$$\frac{7200}{120\,000} = \frac{r(120\,000)}{120\,000}$$
$$0.06 = r$$

Step 4: Express the rate as a percent.

$$0.06 = \frac{6}{100} = 6\%$$

Dan earns a commission of 6%.

4. Rhonda sold a computer for $1500 and earned a commission of $45. What was her rate of commission?

Check your answer by turning to the Appendix.

You can also use the rate of commission to find the amount a salesperson earns.

Example 2

Richard sold a car for $20 000 and earned a 3.5% commission. How much did he earn?

Solution

Method 1: Using a Proportion

Step 1: Write the rate with a second term of 100.

$$3.5\% = \frac{3.5}{100}$$

Step 2: Write a proportion.

$$\frac{\text{commission}}{\text{selling price}} \qquad \frac{3.5}{100} = \frac{\square}{20\,000}$$

Step 3: Find the missing value.

$$\frac{\text{commission}}{\text{selling price}} \qquad \frac{3.5}{100} \overset{\times 200}{\underset{\times 200}{=}} \frac{700}{20\,000}$$

Richard earned $700 in commission.

Method 2: Using a Formula

Step 1: Write a formula.

Let c be the commission (in dollars).
Let r be the rate of commission (as a decimal number).
Let s be the selling price (in dollars).

$$c = rs$$

Step 2: Write the rate as a decimal number.

$$3.5\% = 3.5 \div 100 = 0.035$$

Step 3: Substitute the given values into the formula.

$$\begin{aligned} c &= rs \\ &= 0.035(20\,000) \\ &= 700 \end{aligned}$$

Richard earned $700 in commission.

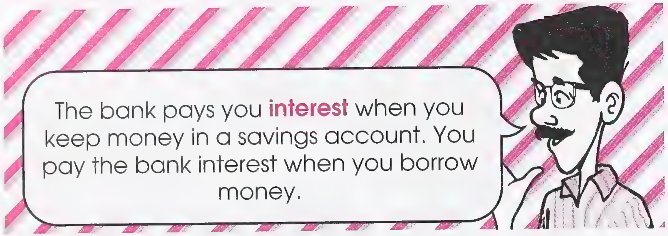
Use a calculator to answer questions 5 and 6.

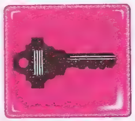
5. Mr. Ristoff sold $3500 of furniture and earned 3% commission. How much money did he earn?

6. Ms. Carlos sold $15 000 of investments and earned 1.25% commission. How much money did she earn?

Check your answers by turning to the Appendix.

Interest





Interest is the amount paid for the use of money.

The rate of interest is expressed as a percent.

The money which is invested or borrowed is the **principal**.

Simple interest can be calculated by the following formula, where I is the interest (in dollars), P is the principal (in dollars), r is the interest rate per year (expressed as a decimal), and t is the length of time (in years).

$$I = Prt$$

Example 1

Mrs. Cartwright deposited \$3000 in a savings account for one year. If the bank pays her simple interest of $3\frac{1}{2}\%$ per year, how much interest did Mrs. Cartwright earn?

Solution

Step 1: Write the percent as a decimal number or a fraction.

$$\begin{aligned} 3\frac{1}{2}\% &= 3.5\% \\ &= 3.5 \div 100 \\ &= 0.035 \end{aligned}$$

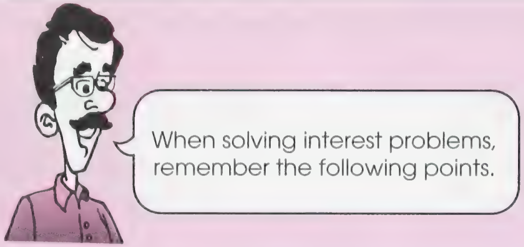
Step 2: Calculate the interest.

$$\begin{aligned} I &= Prt \\ &= 3000(0.035)(1) \\ &= 105 \end{aligned}$$

Mrs. Cartwright earned \$105 in interest.

7. Mr. Marston borrowed \$14 000 from the bank for one year. If the bank charged him simple interest of $12\frac{1}{4}\%$ on the loan, how much interest did Mr. Marston pay?

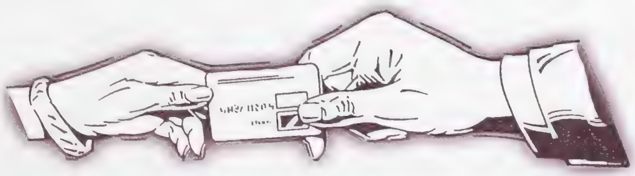
Check your answer by turning to the Appendix.



- In the simple interest formula, the interest rate is expressed as a percent per annum (%/a).
- If the time is given in months or days, you must first change the time to a fraction of a year.
- If interest is given as a monthly rate or a daily rate, you should also change it to an annual rate.

Example 2

A certain department store charges simple interest of $2\frac{1}{2}\%$ per month on its credit card.



Rudy owed \$300 for 8 months on his credit card. Calculate the interest he paid.

Solution

Step 1: Express the time in years.

$$\begin{aligned} 1 \text{ mo} &= \frac{1}{12} \text{ a} \\ \therefore\ 8 \text{ mo} &= \frac{8}{12} \text{ a} \\ &= \frac{2}{3} \text{ a} \end{aligned}$$

Step 2: Express the percent as %/a.

$$\begin{aligned} 1\%/\text{mo} &= 12\%/\text{a} \\ \therefore\ 2\frac{1}{2}\%/\text{mo} &= 30\%/\text{a} \end{aligned}$$

Step 3: Express the %/a as a decimal number.

$$\begin{aligned} 30\% &= 30 \div 100 \\ &= 0.3 \end{aligned}$$

Step 4: Calculate the interest.

$$\begin{aligned} I &= Prt \\ &= 300(0.3)\left(\frac{2}{3}\right) \\ &= 60 \end{aligned}$$

Rudy paid \$60 in interest.

Use a calculator to answer question 8.

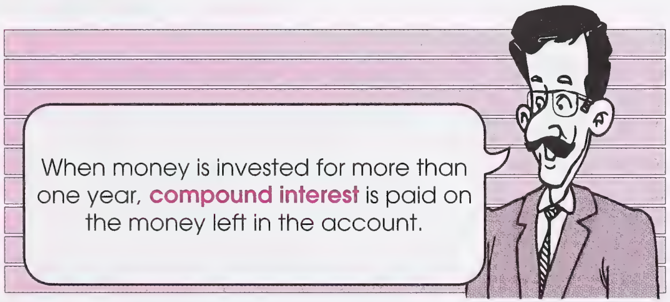
8. Find the simple interest in each of the following situations.

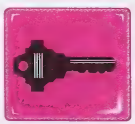
  a. \$3000 borrowed at 5%/a for three months

  b. \$450 borrowed at $1\frac{1}{2}$%/mo for one year

  c. \$1500 borrowed at 2%/mo for six months

Check your answers by turning to the Appendix.





Compound interest is the interest earned (charged) on an amount of money and added to the principal to earn (charge) more interest in the following year.

Example 3

Mr. Lee invested \$1500 at a compound rate of 12% per year. How much money will he have after six years?

Solution

You can use a spreadsheet to calculate the amount. These are the data and formulas you enter.

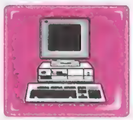
	A	B	C
1	End of Year	Interest	Amount
2	1	=1500*0.12	=1500+B2
3	2	=C2*0.12	=C2+B3
4	3	=C3*0.12	=C3+B4
5	4	=C4*0.12	=C4+B5
6	5	=C5*0.12	=C5+B6
7	6	=C6*0.12	=C6+B7

This is the result. (**Note:** Dollar amounts have been rounded to the nearest cent.)

	A	B	C
1	End of Year	Interest	Amount
2	1	$180.00	$1680.00
3	2	$201.60	$1881.60
4	3	$225.79	$2107.39
5	4	$252.89	$2360.28
6	5	$283.23	$2643.51
7	6	$317.22	$2960.73

At the end of six years, Phil will have $2960.73.

9. a. Why was the formula =1500*0.12 entered in cell B2?
b. Why was the formula =1500+B2 entered in cell C2?
c. Why was the formula =C2*0.12 entered in cell B3?
d. Why was the formula =C2+B3 entered in cell C3?



Use a computer and a spreadsheet program to answer question 10. If you do not have access to a computer, you may create a chart by hand and do the calculations with the help of a calculator.

10. Mrs. Kowalski invested $1500 at a compound rate of 18% per year. How much will she have after six years?

Check your answers by turning to the Appendix.

Taxes and Discounts

In Western Canada a goods and services tax (GST) and provincial sales tax (PST) may be added to the purchase price of most items you buy.

The following chart shows the amount of GST and PST charged in the various western provinces and the territories.

Province/Territory	PST (%)*	GST (%)*
Manitoba	7	7
Saskatchewan	7	7
Alberta	0	7
British Columbia	6.5	7
Northwest Territories	0	7
Yukon	0	7

*These are the percentages in effect at the time of the publication of this course.

Example 1

A pair of runners is priced at $45. What would be the total cost, including GST and PST, of the runners in British Columbia?

Step 1: Calculate the GST.

$$7\% \text{ of } 45 = 0.07 \times 45$$
$$= 3.15$$

$$7\% = 7 \div 100$$
$$= 0.07$$

The GST is $3.15.

Step 2: Calculate the PST.

$$6.5\% \text{ of } 45 = 0.065 \times 45$$
$$= 2.93$$

$$6.5\% = 6.5 \div 100$$
$$= 0.065$$

The PST is $2.93.

Step 3: Find the total cost.

$$\begin{array}{r} 45.00 \\ 3.15 \\ +\quad 2.93 \\ \hline 51.08 \end{array}$$

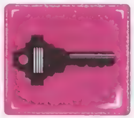
The total cost of the runners in British Columbia is $51.08.

Use a calculator to answer question 11.

11. A snowboard costs $600. Use the chart on the previous page to calculate the total cost, including the GST and PST, of the snowboard in each of the following provinces:

a. Alberta **b.** Manitoba **c.** British Columbia

Check your answers by turning to the Appendix.



A discount is the amount by which the regular price is reduced. The discount rate is usually expressed as a percent.

Example 2

A store has a 5%-off sale on video games. The regular price of a game is $60.

What is the sale price of the game, not including tax?

Solution

Step 1: Express the discount rate as a decimal.

$$\begin{aligned} 5\% &= 5 \div 100 \\ &= 0.05 \end{aligned}$$

Step 2: Calculate the discount in dollars.

$$\begin{aligned} 5\% \text{ of } 60 &= 0.05 \times 60 \\ &= 3 \end{aligned}$$

The discount is $3.

Step 3: Calculate the sale price.

$$60 - 3 = 57$$

The sale price of the video game, not including tax, is $57.

12. A tire that has a regular price of $80 is on sale for 25% off.

What is the sale price, not including tax?

13. A department store has reduced the cost of coats by 12.5%.

If a coat is regularly priced at $240, what is the sale price?

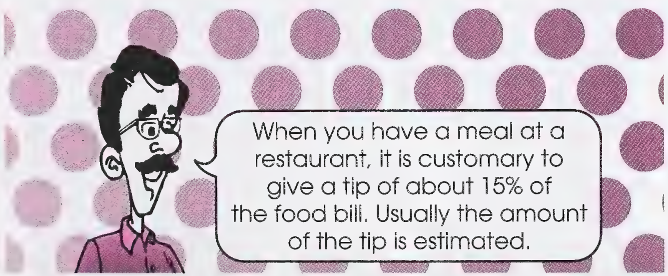
Use a computer and a spreadsheet program to answer question 14. If you do not have access to a computer, you may create a chart by hand and do the calculations with the help of a calculator.

14. A merchant put a stereo set on sale at a 10% discount. The next week, the merchant reduced the sale price by another 10%. The next week, the merchant reduced the sale price by 10% again. The following week, the merchant took off another 10%.

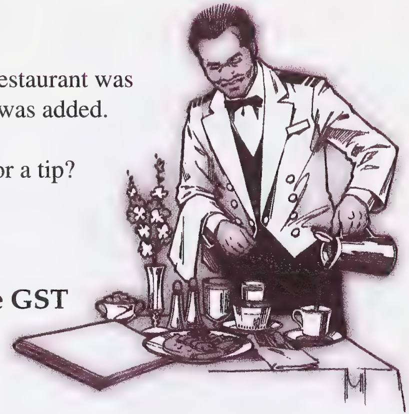
If the stereo was regularly priced at $1500, what was the final sale price?

Check your answers by turning to the Appendix.

Tipping



Example



The food bill for a meal at a restaurant was $14 before the GST of $0.98 was added.

How much should be given for a tip?

Solution

Method 1: Doubling the GST

A quick way to estimate the tip is to double the GST.

$$2 \times 0.98 \doteq 2$$

A tip of about $2 should be given.

Method 2: Rounding Up and Down

Rounding Down

$14 \doteq 10$

$$\begin{aligned} &15\% \text{ of } 10 \\ &= 0.15 \times 10 \\ &= 1.50 \end{aligned}$$

Rounding Up

$14 \doteq 20$

$$\begin{aligned} &15\% \text{ of } 20 \\ &= 0.15 \times 20 \\ &= 3.00 \end{aligned}$$

A tip that is between $1.50 and $3.00 should be given.

15. Ms. Caldwell and Ms. Ruhl ate in a restaurant. The total food bill was \$35. The GST of \$2.45 was added to this bill.

How much of a tip should the ladies give?

Check your answer by turning to the Appendix.

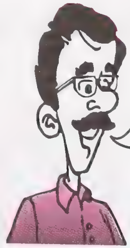
Use the Internet search engines to get some tips on tipping. Whom do you tip besides food servers? When should you tip? How much should you tip?

Now Try This

Use a problem-solving strategy to answer the following question.

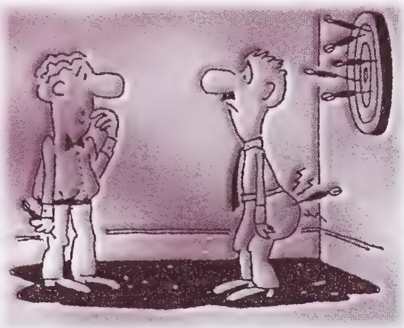
16. Mr. Haidar baked a batch of cookies which he divided among his four children. He gave 30% of the cookies to Astrid, 25% of the cookies to Jessie, 20% of the cookies to Bernard, and 5 cookies to Clayton. How many cookies did Mr. Haidar bake?

Check your answer by turning to the Appendix.



In this activity you solved percent problems involving commission, interest, tax, discounts, and tipping. You worked with fractional percents and percents greater than 100%. You continued to use the Internet and spreadsheets and you solved another non-routine problem.

Activity 4: Proportion and Scale



Have you ever noticed that many cartoon characters are out of proportion? The size of their heads is often too large for the size of their bodies.

In the preceding cartoon, the ratio of each character's height to the length of his head is about 3 to 1. The actual ratio of a man's height to the length of his head is actually 7.5 to 1.

Proportion is important when making **scale models**.

A scale model is a likeness of an object that has the same proportions as the object.

Gerry and Bob Bell, the owners and operators of the Old Strathcona Model Toy Museum in Edmonton, Alberta, have many scale models.

[1]

Gerry Bell used proportion to design and construct this scale model of the RCMP Musical Ride.

You can use proportional reasoning when you work with models.

[1] Brian Gavriloff, photo, *The Edmonton Journal*, July 1996. Reprinted by permission.

Example 1

Gerry Bell's model of the Musical Ride was constructed on a scale of 1 to 12. If each model horse and rider is 18 cm high, what is the height (in metres) of an actual horse and rider?

Solution

Step 1: Write a proportion.

model:actual

$$\begin{aligned} &1 : 12 \\ &= 18 : \boxed{} \end{aligned}$$

Step 2: Calculate the height of an actual horse and rider in centimetres.

model:actual

$$\begin{array}{ccc} 1 & : & 12 \\ \downarrow \times 18 & & \downarrow \times 18 \\ = 18 & : & \boxed{216} \end{array}$$

The actual horse and rider would be 216 cm in height.

Step 3: Calculate the height of an actual horse and rider in metres.

$100 \text{ cm} = 1 \text{ m}$

$\therefore\ 216 \text{ cm} = 2.16 \text{ m}$

The actual horse and rider would be about 2.16 m in height.

INTERNET

You may wish to visit the Old Strathcona Model & Toy Museum in person (8603-104 Street, Edmonton) or on the Internet. This is the URL of their site:

http://www.discoveredmonton.com/ToyMuseum/

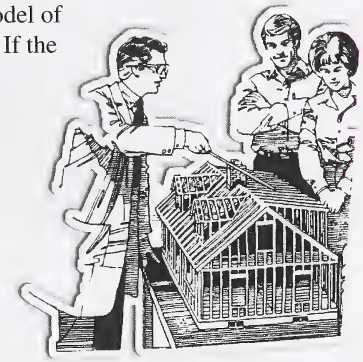
1. The Alberta Provincial Museum had scale models of insects at their exhibit, BugWorld. The scale for the model of the praying mantis was 60 to 1. If the length of the model was 4.5 m, what is the length (in centimetres) of a real praying mantis?


[1]

[1] Bruce Edwards, photo, *The Edmonton Journal*, 13 October 1996, G8. Reprinted by permission.

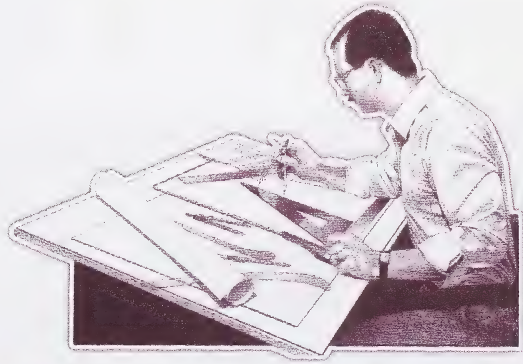
2. Thea has two scale models of tractors. The scale of the larger model is 1 to 16. If the diameter of the large tire on this model tractor is 11.5 cm, what is the diameter (in metres) of the actual tire?

3. An architect made a scale model of a house. The scale is 1 to 24. If the length of the model is 60 cm, what is the actual length (in metres) of the house?



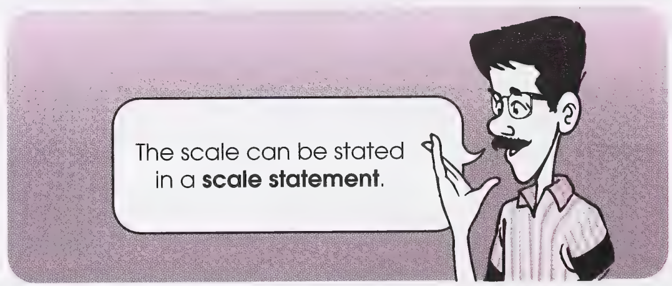
Check your answers by turning to the Appendix.

Technical artists and cartographers (people who create maps) make **scale drawings**.



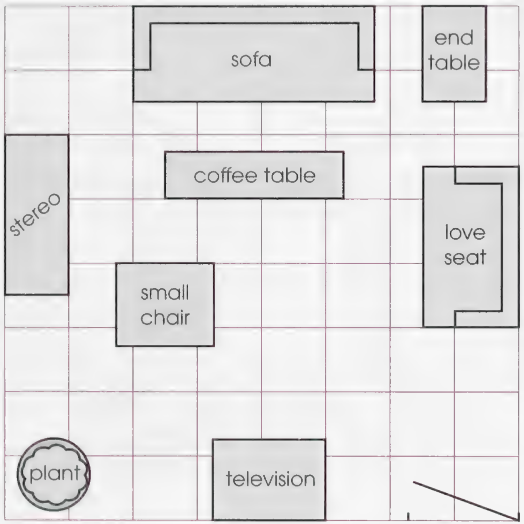
A scale drawing is used to accurately picture a person, animal, or thing that is too large or too small to be drawn its actual size.

The scale may be stated in different ways.



Example 2

An interior designer made this floor plan of Linda's living room. The interior designer used 1 cm × 1 cm grid paper to make the floor plan.



Scale 1 cm represents 50 cm.

Use the scale statement to calculate the actual length (in metres) of the sofa.

Solution

Step 1: Measure the length of the sofa in the drawing.

The length of the sofa in the drawing is 3.7 cm.

Step 2: Write a proportion.

$$\frac{\text{drawn}}{\text{actual}} \qquad \frac{1}{50} = \frac{3.7}{\square}$$

Step 3: Calculate the actual length of the sofa in centimetres.

$$\frac{\text{drawn}}{\text{actual}} \qquad \frac{1}{50} \underset{\times 3.7}{\overset{\times 3.7}{=}} \frac{3.7}{185}$$

The actual length of the sofa is 185 cm.

Step 4: Calculate the length of the sofa in metres.

100 cm = 1 m

∴ 185 cm = 1.85 m

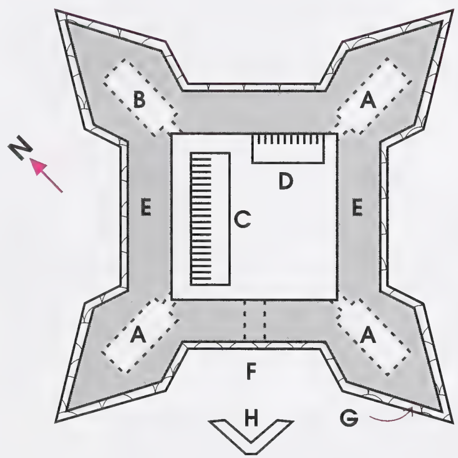
The actual length of the sofa is 1.85 m.

Example 3

This is a scale drawing of the overhead view of an 18th century fur trading fort in Churchill, Manitoba.

The Prince of Wales Fort, About 1770

A	Storehouses	E	Earth wall
B	Magazine	F	Gate
C	Dwelling house	G	Stone parapet
D	Offices	H	Ravelin to protect gate



Scale 1 cm represents 15 m.

Use the scale statement to calculate the length of the dwelling house.

Solution

Step 1: Measure the length of the dwelling house in the drawing.

The length of the dwelling house in the drawing is 2 cm.

Step 2: Write a proportion.

drawn (cm):actual (m)

$$\begin{aligned} &1 : 15 \\ = &2 : \end{aligned}$$

Step 3: Calculate the length of the actual dwelling house.

drawn (cm):actual (m)

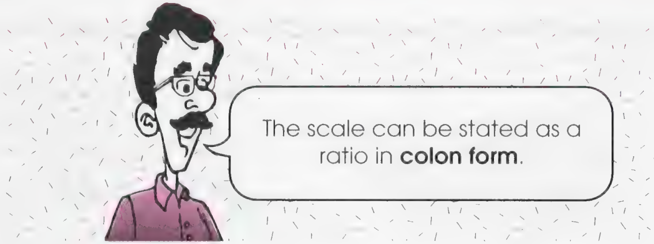
$$\begin{aligned} &1 : 15 \\ &\times 2 \quad \times 2 \\ = &2 : 30 \end{aligned}$$

The length of the actual dwelling house is 30 m.

4. Calculate the actual dimensions (in metres) of the television in Example 2.

5. Calculate the actual width of the gate in Example 3.

Check your answer by turning to the Appendix.



Example 4

Following are scale drawings of the Calgary Tower and the Eiffel Tower. What is the actual height (in metres) of the Eiffel Tower?

Calgary Tower

Scale 1:6400

Eiffel Tower

Scale 1:10 000

Solution

Step 1: Measure the height of the drawn Eiffel Tower.

The height is 3 cm.

Step 2: Write a proportion.

drawn:actual

$$1:10\,000$$
$$= 3:\ \boxed{}$$

Step 3: Calculate the height of the actual Eiffel Tower in centimetres.

drawn:actual

$$\begin{aligned} &1 : 10\,000 \\ &\times 3 \quad \times 3 \\ &= 3 : \boxed{30\,000} \end{aligned}$$

The actual height of the Eiffel Tower is 30 000 cm.

Step 4: Calculate the actual height in metres.

$$100 \text{ cm} = 1 \text{ m}$$
$$\therefore\ 30\,000 \text{ cm} = 300 \text{ m}$$

The actual height of the Eiffel Tower is 300 m.

6. Use the scale ratio in Example 4 to calculate the actual height (in metres) of the Calgary Tower.

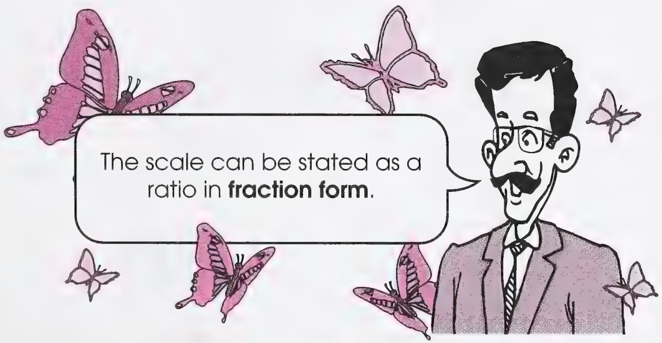
Check your answer by turning to the Appendix.

You may use the Internet to find out more about the actual objects drawn to scale in examples 3 and 4. Visit the Calgary Tower home page at the following uniform resource locator (URL):

http://www.lexicom.ab.ca/~calgary_tower/

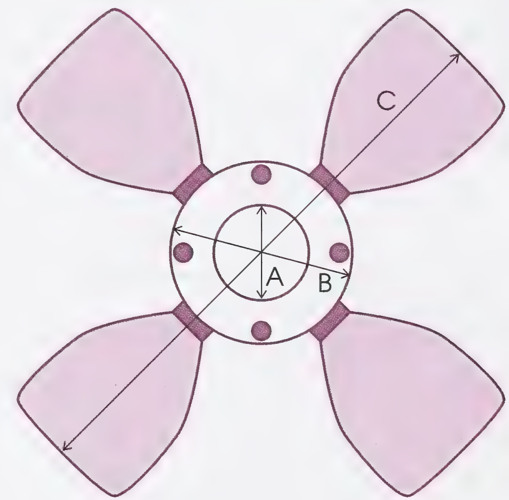
Visit the Prince of Wales Fort National Historic Site at the following uniform resource locator (URL):

http://parkscanada.pch.gc.ca/parks/alphae.htm



Example 5

This is a scale drawing of an automobile fan.



Scale $\frac{1}{10}$ Actual Size

Use the scale ratio to calculate the actual diameter of A.

Solution

Step 1: Measure the diameter of A in the drawing.

The diameter of A in the drawing is 1.5 cm.

Step 2: Write a proportion.

$$\frac{\text{drawn}}{\text{actual}} \qquad \frac{1}{10} = \frac{1.5}{\square}$$

Step 3: Calculate the actual diameter of A.

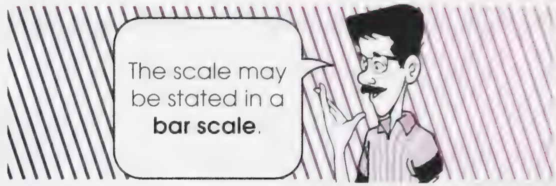
$$\frac{\text{drawn}}{\text{actual}} \qquad \frac{1}{10} \overset{\times 1.5}{\underset{\times 1.5}{=}} \frac{1.5}{15}$$

The actual diameter of A is 15 cm.

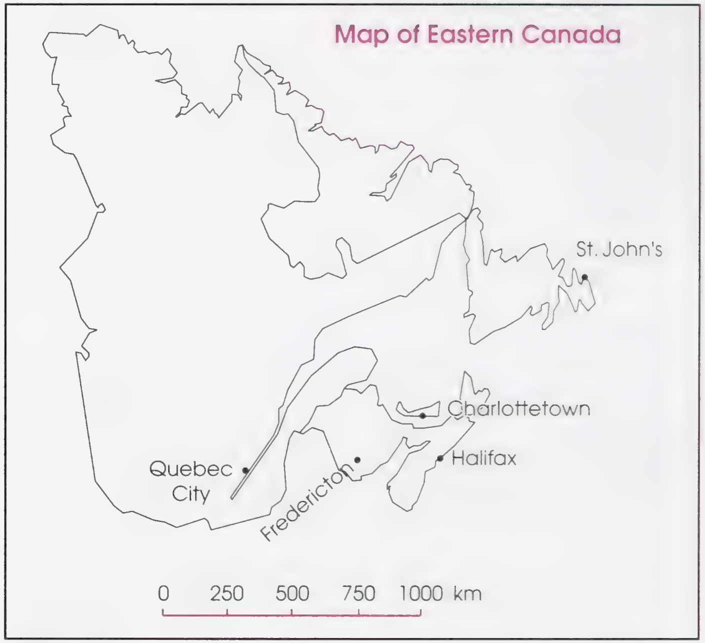
7. Use the scale ratio in Example 5 to calculate the actual diameter of B.

8. Use the scale ratio in Example 5 to calculate the actual length of C.

Check your answers by turning to the Appendix.



Example 6



This map of Eastern Canada is drawn to scale. Use the bar scale to calculate the straight-line distance from St. John's to Fredericton.

Solution

Step 1: On the map, measure the straight-line distance (in centimetres) from St. John's to Fredericton.

On the map it is 4.5 cm from St. John's to Fredericton.

Step 2: Use the bar scale to write a scale statement.

Scale 1 cm represents 250 km.

Step 3: Write a proportion.

drawn (cm):actual (km) $\quad 1 : 250$

$= 4.5 : \boxed{}$

Step 4: Calculate the actual straight-line distance in kilometres.

drawn (cm):actual (km)

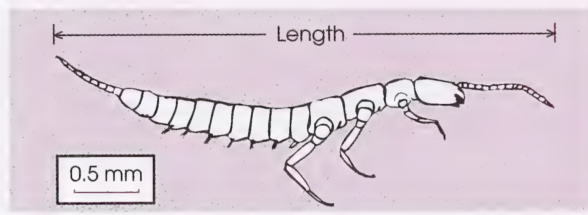
$$\begin{array}{ccc} 1 & : & 250 \\ \downarrow \times 4.5 & & \downarrow \times 4.5 \\ = 4.5 & : & \boxed{1125} \end{array}$$

The actual straight-line distance from St. John's to Fredericton is 1125 km.

9. Use the bar scale in Example 6 to calculate the straight-line distance between these cities.

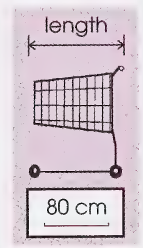
a. Charlottetown and Halifax
b. Quebec City and St. John's

10. a. Following is a scale drawing of an insect. Use the bar scale to calculate the actual length of the insect.

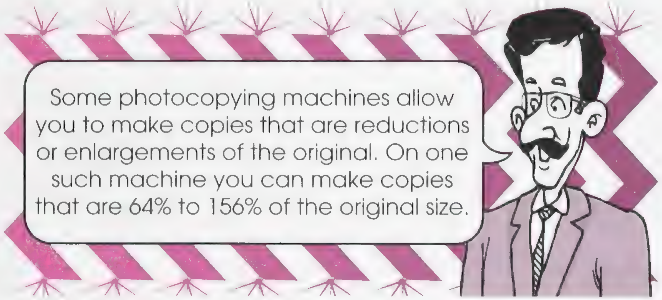


b. Is the scale drawing an enlargement or a reduction?

11. This is a scale drawing of a shopping cart. Use the scale bar to calculate the actual length (in metres) of the shopping cart.



Check your answers by turning to the Appendix.



The numbers 64% and 156% are **scale factors**.

The scale factor is the ratio of the copied length to the original length. A scale factor is expressed as a single number. The number may be a percent, a whole number, or a fraction.

When making an enlargement or a reduction, you can use the scale factor.

Example 7

Helga has a drawing that is $10\ \text{cm} \times 12\ \text{cm}$. If she photocopies the drawing at a setting of 85%, what will be the dimensions of the copy?

Solution

Step 1: Use the scale factor to calculate the width of the reduction.

$$\begin{aligned} 85\% \text{ of } 10 &= 0.85 \times 10 \\ &= 8.5 \end{aligned}$$

The width is 8.5 cm.

Step 2: Use the scale factor to calculate the length of the reduction.

$$\begin{aligned} 85\% \text{ of } 12 &= 0.85 \times 12 \\ &= 10.2 \end{aligned}$$

The length is 10.2 cm.

The dimensions of the copy will be $8.5\ \text{cm} \times 10.2\ \text{cm}$.

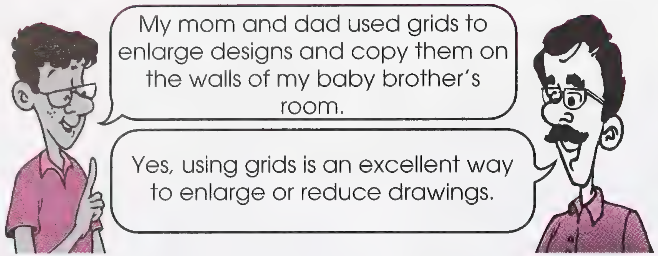
12. Rachel has a photograph that is 6 cm × 4 cm. If she enlarges the photograph at a setting of 130%, what will be its dimensions?

13. Duncan has a drawing that is 8 cm in length.

a. If Duncan wants to make a copy that is 4 cm in length, what setting should he select on the photocopier?

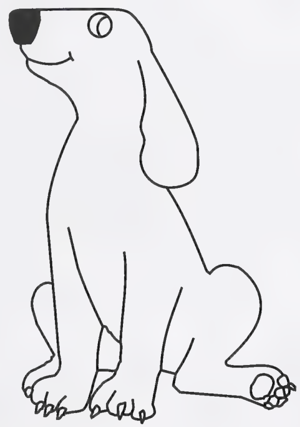
b. If Duncan wants to make a drawing that is 10 cm in length, what setting should he select?

Check your answers by turning to the Appendix.



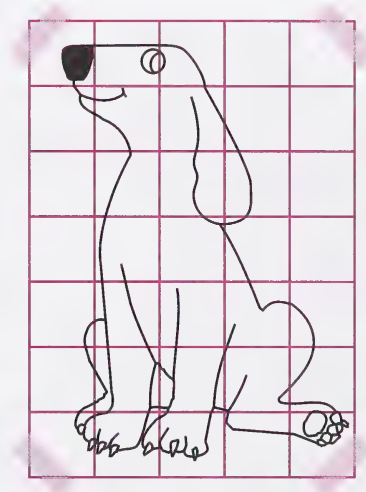
Example 8

Enlarge this drawing of a dog using a scale factor of 2.



Solution

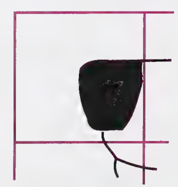
Step 1: Make a grid on tracing paper and cover the drawing. Tape the tracing paper lightly so it doesn't move.



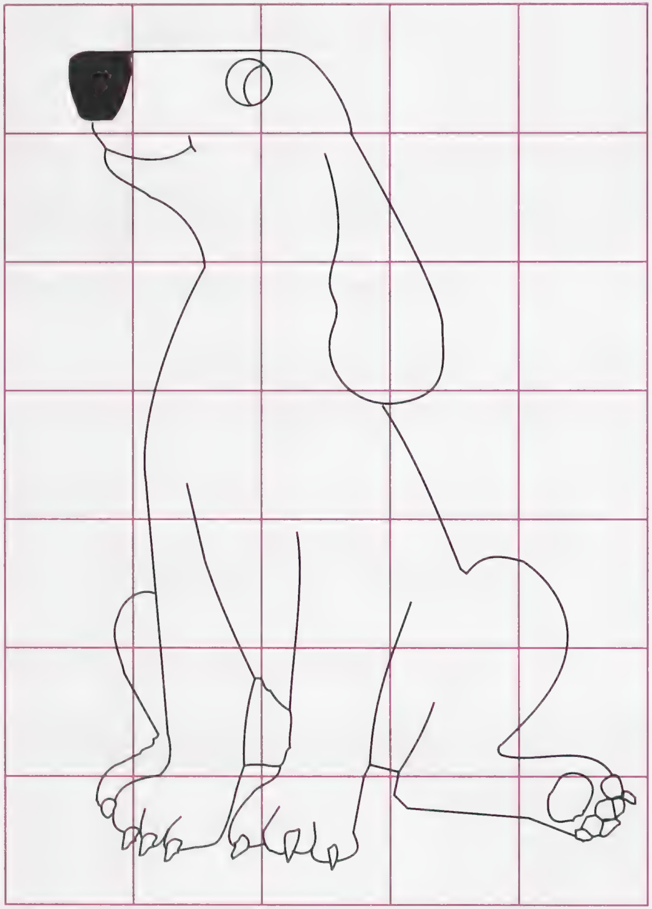
Each square in the grid is 1 cm × 1 cm.

Step 2: Since the scale factor is 2, make a grid of squares with sides twice as long as the grid in Step 1. Each square in the grid will be 2 cm × 2 cm.

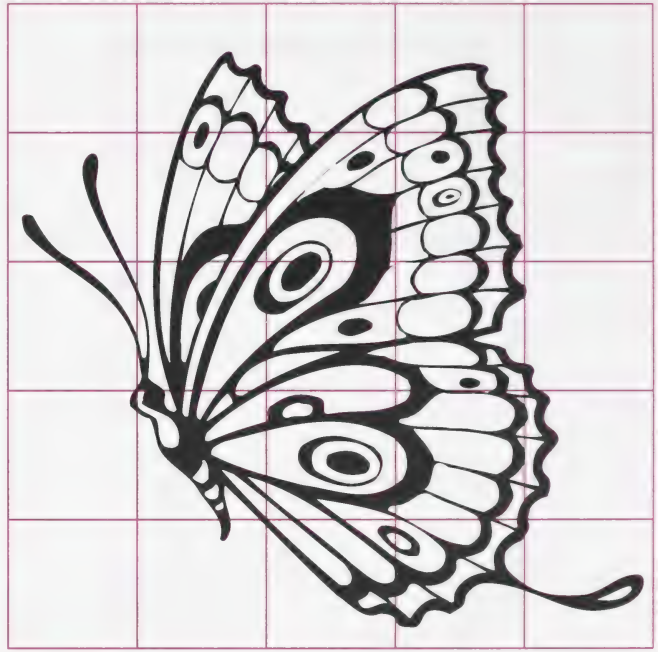
Step 3: Copy the contents of each square in the corresponding square in the grid you have made to produce the enlargement. For example, the contents of the upper left square will look like this:



This is the enlargement of the drawing using a scale factor of 2.



14. Reduce this drawing of a butterfly by copying it onto 1 cm × 1 cm graph paper. The scale factor will be $\frac{1}{2}$.



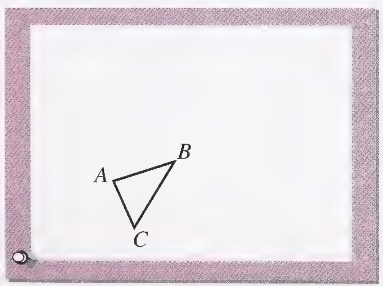
Check your answer by turning to the Appendix.

Example 9

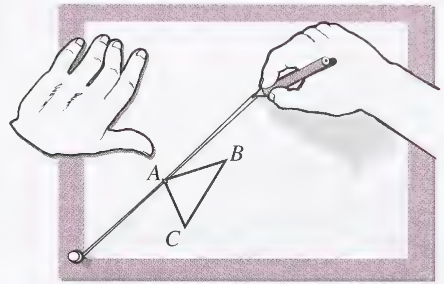
Use a paper fastener, a piece of cardboard, and an elastic band to enlarge a triangle.

Solution

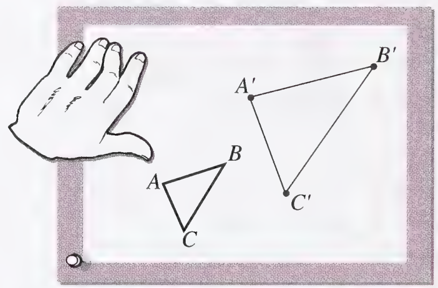
Step 1: Draw a triangle on a piece of paper. Label the triangle ABC. Tape the piece of paper to a piece of cardboard. Push the paper fastener through the cardboard at the bottom left corner of the paper.

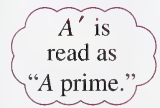


Step 2: Make a knot in the middle of an elastic band. Attach one end of the elastic band to the paper fastener. Put a pen or pencil in the other end of the elastic band. Use the pen to stretch the elastic until the knot is on vertex A of the triangle. Mark the point where the **pencil** is and label the point A'.

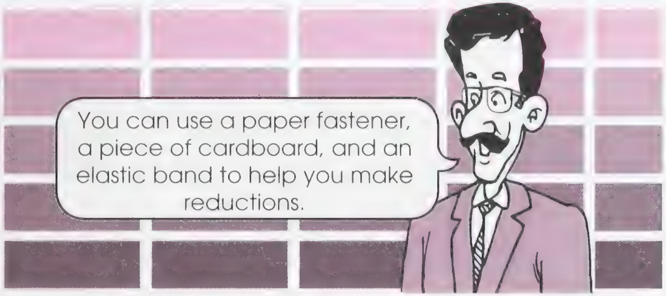


Step 3: Repeat Step 2 until all the vertices of triangle $A'B'C'$ have been marked. Then connect the points of the triangle with line segments.





$\Delta A'B'C'$ is the enlargement of ΔABC.

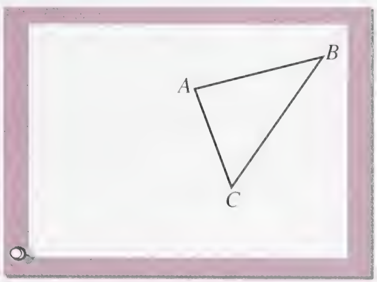


Example 10

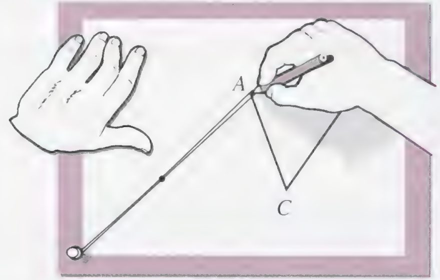
Use a paper fastener, a piece of cardboard, and an elastic band to reduce a triangle.

Solution

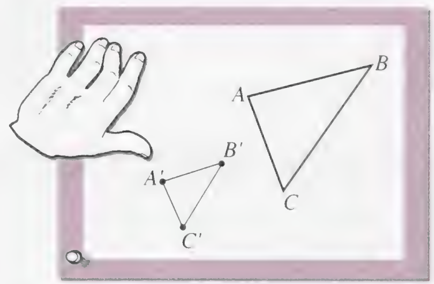
Step 1: Draw a triangle on a piece of paper. Label the triangle *ABC*. Tape the piece of paper to a piece of cardboard. Push the paper fastener through the cardboard at the bottom left corner of the paper.



Step 2: Make a knot in the middle of an elastic band. Attach one end of the elastic band to the paper fastener. Put a pen or pencil in the other end of the elastic band. Use the pen to stretch the elastic until the pencil is on vertex *A* of the triangle. Mark the point where the **knot** is and label the point A'.



Step 3: Repeat Step 2 until all the vertices of triangle $A'B'C'$ have been marked. Then connect the points of the triangle with line segments.



A' is read as "A prime."

$\Delta A'B'C'$ is the reduction of ΔABC.

Note: In Example 9, the line segment $A'B'$ is twice as long as AB, $B'C'$ is twice as long as BC, and $A'C'$ is twice as long as AC. In other words, $\Delta A'B'C'$ is an enlargement of ΔABC with a scale factor of 2.

In Example 10, the line segment $A'B'$ is half as long as AB, $B'C'$ is half as long as BC, and $A'C'$ is half as long as AC. In other words, $\Delta A'B'C'$ is a reduction of ΔABC with a scale factor of $\frac{1}{2}$.

15. Gather three elastic bands (of the same size), a paper fastener, a piece of cardboard, a pen or pencil, a ruler, some tape, and a piece of paper.

Make a knot in each elastic band. (Try to put the knots in a different position on each elastic band.)

Complete Step 1 of examples 9 and 10.

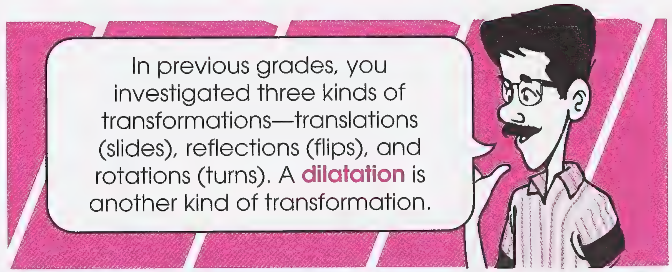
a. Note the position of the knot in one of the elastic bands. Then attach this elastic band to the paper fastener and follow steps 2 and 3 of examples 9 and 10.

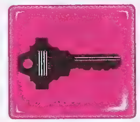
Measure the lengths of the corresponding sides of the original triangle and the drawn triangle. Then calculate the scale factor.

Repeat the process for each of the other elastic bands.

b. What effect does the position of the knot on the elastic have on the scale factor of the drawn triangle?

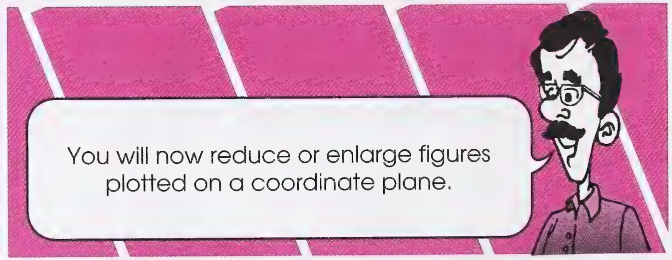
Check your answers by turning to the Appendix.





A dilatation is a kind of transformation in which the image is an enlargement or reduction of the original. The point about which the enlargement or reduction is made is the **dilatation centre**.

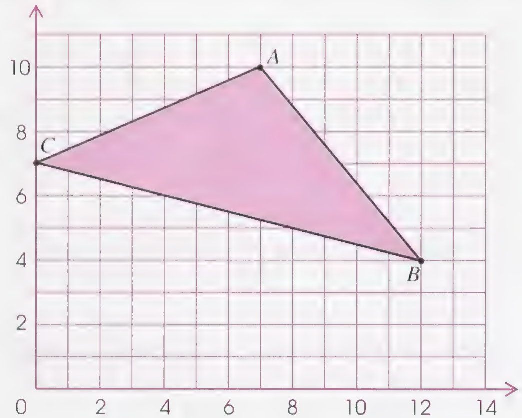
In examples 9 and 10, the triangle $A'B'C'$ is a dilatation of triangle ABC. The paper fastener is the dilatation centre.



When a figure is plotted on a coordinate plane, with the dilatation centre at the origin, the figure can be enlarged or reduced by multiplying the coordinates of each of the vertices of the figure by the scale factor.

Example 11

Reduce ΔABC by a scale factor of $\frac{1}{2}$. The dilatation centre is $(0, 0)$.



Solution

Step 1: Multiply the coordinates of point A by $\frac{1}{2}$ to find A'.

$$\frac{1}{2}\times 7 = 3\frac{1}{2};\ \frac{1}{2}\times 10 = 5$$

Point A has coordinates $(7, 10)$.

$\therefore$ Point A' will have coordinates $\left(3\frac{1}{2}, 5\right)$.

Step 2: Multiply the coordinates of point B by $\frac{1}{2}$ to find B'.

$$\frac{1}{2}\times 12 = 6;\ \frac{1}{2}\times 4 = 2$$

Point B has coordinates $(12, 4)$.

$\therefore$ Point B' will have coordinates $(6, 2)$.

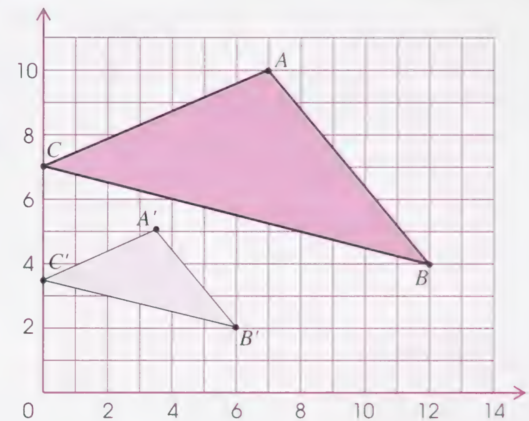
Step 3: Multiply the coordinates of point C by $\frac{1}{2}$ to find C'.

$$\frac{1}{2}\times 0 = 0;\ \frac{1}{2}\times 7 = 3\frac{1}{2}$$

Point C has coordinates $(0, 7)$.

$\therefore$ Point C' will have coordinates $\left(0, 3\frac{1}{2}\right)$.

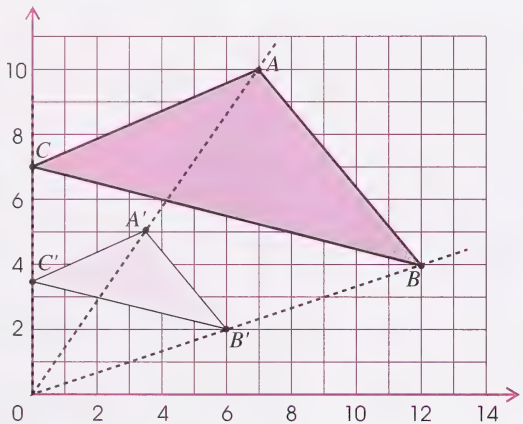
Step 4: Plot A', B', and C'; connect the points with line segments.



$\Delta A'B'C'$ is the reduction of ΔABC.

Note: Because the scale factor is $\frac{1}{2}$, the line segment $A'C'$ is half as long as AC, $A'B'$ is half as long as AB, and $C'B'$ is half as long as CB.

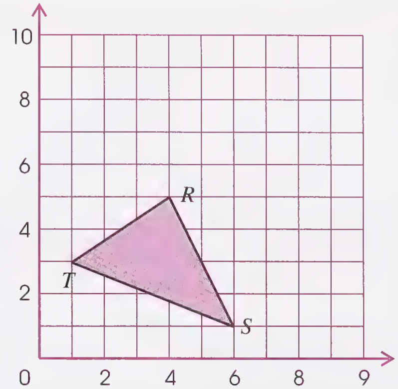
If you connect the dilatation centre and the corresponding vertices of ΔABC and $\Delta A'B'C'$ in Example 11, you will see that A and A' are on the same line. (Also, B and B' are on the same line, and C and C' are on the same line.)



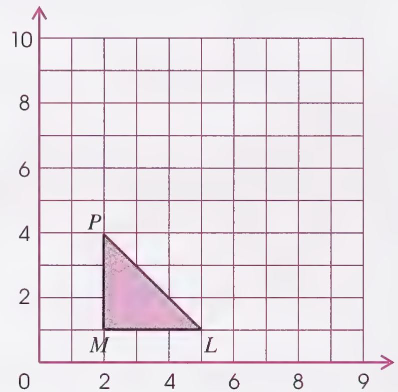
16. Copy each of the following figures onto graph paper. Then enlarge each of the figures by a scale factor of 2. The dilatation centre is $(0, 0)$.

Hint: Use a different sheet of graph paper for each figure. You may have to extend the x-axis and/or the y-axis.

a.



b.



17. Repeat questions 16. a. and 16. b. using a scale factor of $\frac{1}{2}$.

18. Repeat questions 16. a. and 16. b. using a scale factor of 3.

19. In questions 16, 17, and 18, connect the dilatation centre and corresponding vertices with line segments. What do you notice?

Check your answers by turning to the Appendix.

Did You Know?

Have you ever been to an IMAX theatre? IMAX images are projected onto giant screens, up to eight storeys high. The size of the IMAX frame (in the film) is part of the reason for sharp and clear images. What part did Canadians play in the development of the IMAX system?

Find and read the article entitled "The IMAX System" in the Appendix.

20. Examine the following chart which compares the sizes of frames in various films.

Film	Perforations per Frame	Area (mm^2)
Standard 16 mm	1	69.5
Standard 35 mm	4	319
Standard 70 mm	5	1072
IMAX 70 mm	15	3376

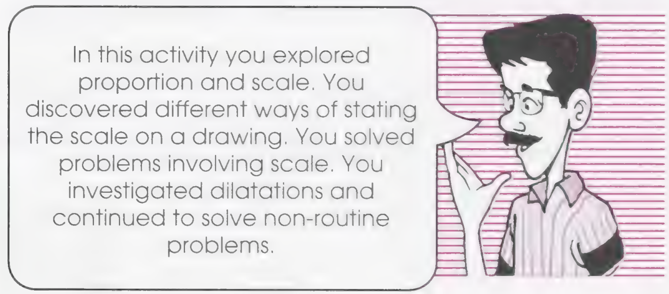
a. How much larger is the IMAX image than a standard 35-mm frame? than a standard 70-mm frame?

b. A typical home movie film is 8 mm. How do you think the quality of a home movie image would compare with an IMAX movie image? Why?

Check your answers by turning to the Appendix.

Explore the Internet for more information about IMAX.

You may also be able to visit an IMAX theatre near you. (In Western Canada, there are IMAX theatres in Vancouver, Calgary, Edmonton, and Winnipeg.)



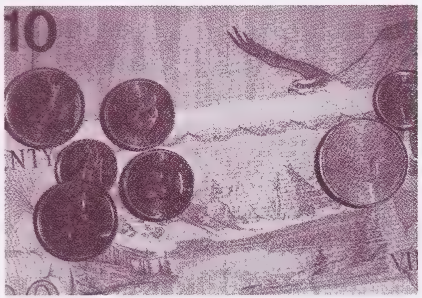
Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

Many calculators have a percent key. This key is helpful when you are finding a percent of a number.

Example 1



Fred sells computers. He works on commission and earns 2.5% of the purchase price. If he sells a computer for $1800, what is his commission?

Solution

Enter the numbers to be multiplied first and the percent key last. It is not necessary to press the equals key.

1 8 0 0 × 2 • 5 %

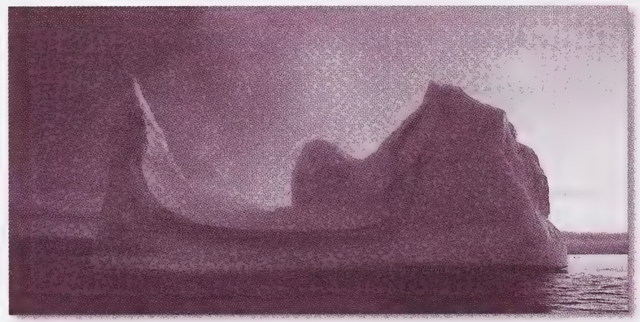
45.

Fred's commission is $45.

Note: Sometimes the second function or shift key must be pressed before the percent key. Check your Owner's Manual to determine how your calculator works.

Use the percent key on a calculator to answer questions 1 to 4.

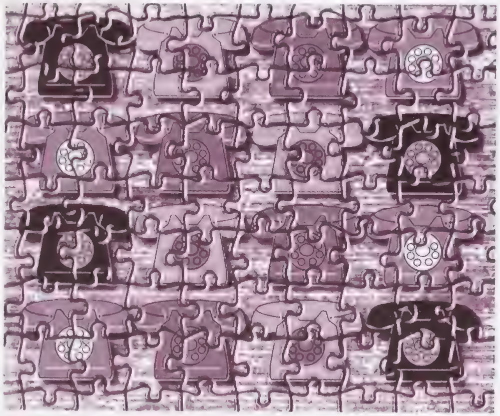
1. Icebergs are dangerous because most of their mass is below the water where it cannot be seen. A particular iceberg measures 300 m from top to bottom and 75% of it is below sea level. What is the depth of the iceberg below the water?



2. Helmut's expenses were 250% of Carrie's expenses. If Carrie's expenses were $128, what were Helmut's expenses?

3. Milk is 87.5% water. In 500 mL of milk, how much water is there?

4. In a shipment of telephones, 2.5% of the telephones were black. If there were 400 telephones in the shipment, how many were black?



Check your answers by turning to the Appendix.

Example 2

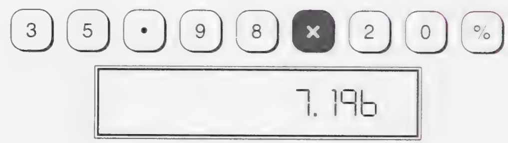
A store is holding a 20%-off sale on all men's and women's moccasins and women's flat shoes. Calculate the sale price for flats regularly priced $35.98.

PHOTO SEARCH LTD.

Solution

Method 1: Using Two Steps on the Calculator

Step 1: Calculate the discount.



The discount is $7.20.

Step 2: Calculate the sale price.

3 5 • 9 8 − 7 • 2 0 =

28.78

The sale price is $28.78.

Method 2: Using One Step on the Calculator

Taking 20% off means that you pay 80% of the regular price.

100% – 20% = 80%

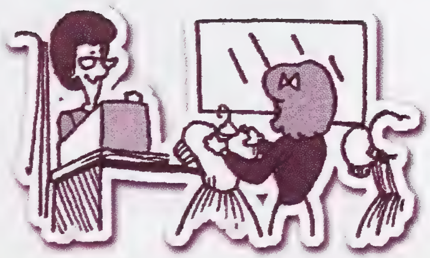
3 5 • 9 8 × 8 0 %

28.784

The sale price is $28.78.

Example 3

The price of a dress is $49.98. What is the total price of the dress, including the GST of 7%?



Solution

Method 1: Using Two Steps on the Calculator

Step 1: Find the amount of tax.

4 9 • 9 8 × 7 %

3.4986

The tax is $3.50.

Step 2: Find the total price.

4 9 • 9 8 + 3 • 5 0 =

53.48

The total price is $53.48.

Method 2: Using One Step on the Calculator

Adding 7% onto the price means that you pay 107% of the price.

100% + 7% = 107%

4 9 • 9 8 × 1 0 7 %

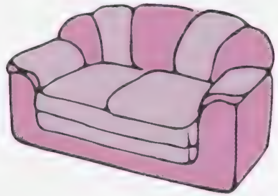
53.4786

The total price is $53.48.

Use the percent key on a calculator to answer questions 5 and 6.

5. Calculate the sale price for each of the following items:

a. A sofa that regularly costs \$950 is on sale at 25% off.

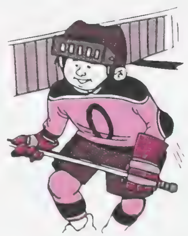


b. A microwave oven that regularly costs \$250 is on sale at 20% off.

6. Calculate the cost, including the GST (of 7%), for each of the following items:

a. a skateboard that costs \$90

b. a hockey jersey that costs \$50



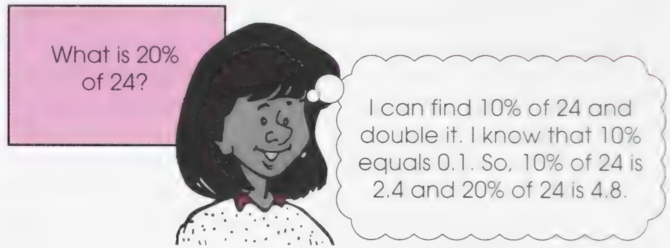
Check your answers by turning to the Appendix.

Enrichment

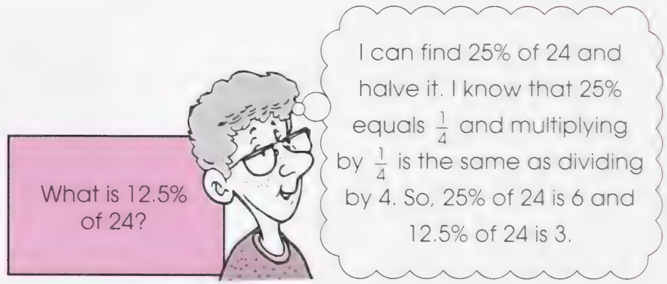
In this Enrichment you will improve your ability to mentally compute a percent of a number.

When doing mental math, it is sometimes easier to work with the decimal form of the percent. At other times it is easier to work with the fraction form.

Example 1



Example 2



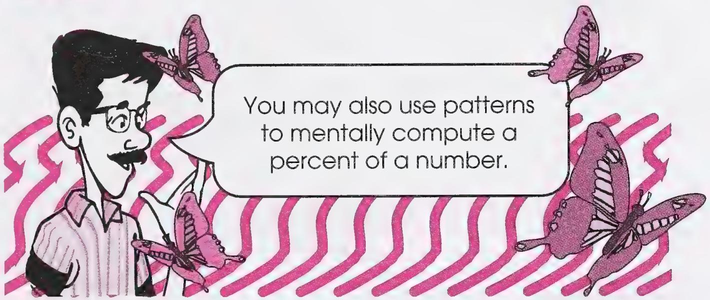
1. Mentally compute each of the following.

   a. 80% of 80 **b.** 30% of 40

2. Mentally compute each of the following.

   a. $33\frac{1}{3}$% of 60 **b.** $66\frac{2}{3}$% of 90
   c. 25% of 40 **d.** 2.5% of 80

Check your answers by turning to the Appendix.



Use a calculator to answer question 3.

3. Compute each of the following.

   a. 18% of 50 **b.** 50% of 18
   c. 88% of 25 **d.** 25% of 88

4. What pattern did you notice in question 3?

5. Apply the pattern you discovered in question 4 to mentally compute each of the following.

   a. 26% of 50 **b.** 84% of 25
   c. 55% of 20 **d.** 28% of 50

Use a calculator to answer question 6.

6. Compute each of the following.

   a. 45% of 60 **b.** 20% of 60 + 25% of 60
   c. 19% of 25 **d.** 20% of 25 – 1% of 25

7. What pattern did you notice in question 6?

8. Apply the pattern you discovered in question 7 to mentally compute each of the following.

   a. 35% of 80 **b.** 31% of 60
   c. 79% of 50 **d.** 19% of 40

Check your answers by turning to the Appendix.

Conclusion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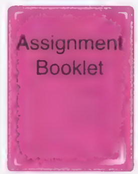
In this section you solved problems involving ratios, rates, and percents. You used three-term ratios, unit rates, and fractional percents. You investigated many situations in which ratios, rates, and percents are used—commissions, simple interest, compound interest, taxes, and discounts. You gained a deeper appreciation for proportion and scale. You explored scale models and scale drawings.

Ratios, rates, and percents are frequently used in describing and comparing. For example, sheep bred for their wool account for 50% of the world sheep population. A fine-wool sheep such as a merino can have 9000 wool follicles per square centimetre of skin. There are about 100 000 000 fibres per fleece. A merino can grow 8 900 000 m of wool fibre per year. After cleaning and processing the wool fibre, a merino provides about 7000 m of yarn per year.

If about 1400 m of yarn is required to make one medium-sized sweater, how many sweaters can be made from the fleece of one merino each year?

Assignment



You are now ready to complete the assignment for Section 1.

Section 2: Working with Powers and Roots

[1]

Do you think that insects are creepy or fascinating? Here are some amazing facts about bugs. Experts estimate that there are between 5 million and 15 million species of insects in the world. Some insects, such as the Australian burrowing cockroach (shown in the photograph), are quite large. Other insects are extremely tiny. Did you know that you have teeny follicle mites on your eyelashes? Everyone does. These microscopic insects feed on dead skin.

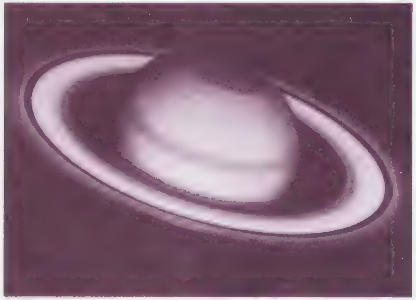
Powers can be used to describe large numbers, such as the number of insects in the world, and small measurements, such as the size of follicle mites.

In this section you will write numbers in scientific notation. You will investigate the relationship between squares and square roots. You will discover the relation between the sides of a right triangle—the Pythagorean relation.

[1] Bruce Edwards, photograph, *The Edmonton Journal*, 13 October 1996, G8. Reprinted by permission.

Activity 1: Scientific Notation

Satellite-based telescopes such as the Hubble Space Telescope have enabled astronomers to see distant planets and moons more clearly than ever before. For example, this image of Saturn was taken by the Hubble Space Telescope at a distance of 1 387 000 000 km from Earth.



NASA

Electron microscopes have allowed scientists to view tiny organisms and cell structures in greater detail than ever before. For example, under a microscope, a white blood cell that has a length of 0.012 mm can be examined. This image shows a white blood cell in between red blood cells.

[1]

In this activity you will work with very large and very small numbers.

[1] Reproduced by permission from <http://ike.engr.washington.edu/software/eduimg/em/img0032.jpg> (7 July 1997). Copyright 1997 by International Business Machines Corporation.

Large Numbers

If you began counting and you named a different number every second without stopping, how long would it take to count to one million? ten million? one hundred million? one billion?

- To count to one million, it would take nearly 12 days.
- To count to ten million, it would take almost 116 days.
- To count to one hundred million, it would take over 3 years.
- To count to one billion, it would take almost 32 years.

One million, ten million, one hundred million, and one billion can each be written in **standard form** or as a **power of ten**.

Standard form is the usual form of a number. Powers with a base of ten are called powers of ten.

Name	Standard Form	Power of Ten
one million	1 000 000	10^6
ten million	10 000 000	10^7
one hundred million	100 000 000	10^8
one billion	1 000 000 000	10^9

$$\begin{aligned} 10^6 &= 10\times10\times10\times10\times10\times10 \\ &= 1\,000\,000 \end{aligned}$$

1. Each number in the following sequence is ten times as large as the number before it.

   10, 100, 1000, 10 000, 100 000, ___, ___, ___, …

   a. Use patterns to find the next three numbers in the sequence.

   b. Use patterns to find the number preceding 10 in the sequence.

2. The sequence in question 1 can be written as powers of ten.

   10^1, 10^2, 10^3, 10^4, 10^5, ___, ___, ___, …

   a. Use patterns to find the next three numbers in the sequence.

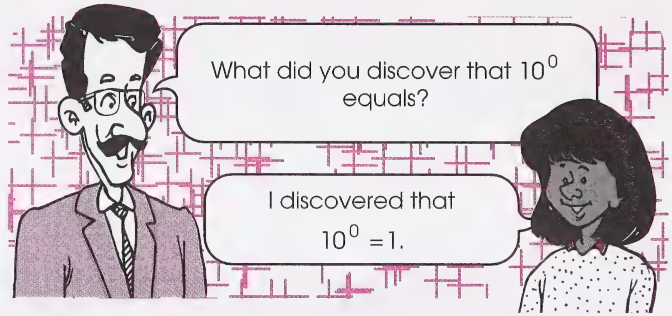
   b. Use patterns to find the power preceding 10^1 in the sequence.

3. Examine the corresponding forms of the numbers in the sequences in questions 1 and 2.

   a. What do you notice about each power of ten and its standard form?

   b. What does 10^0 equal?

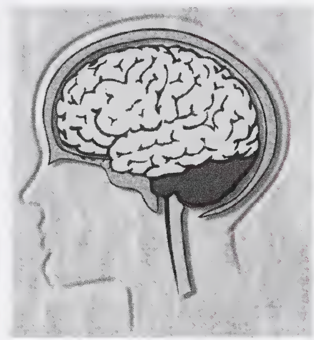
Check your answers by turning to the Appendix.



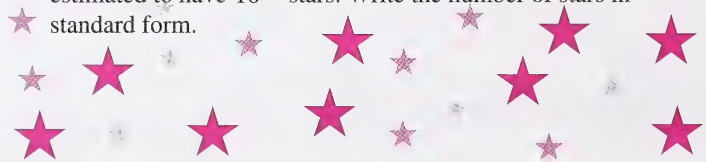
Answer questions 4 to 7 to review working with powers of ten.

4. The human brain contains about 10 000 000 000 cells called neurons.

   Write the number of cells as a power of ten.



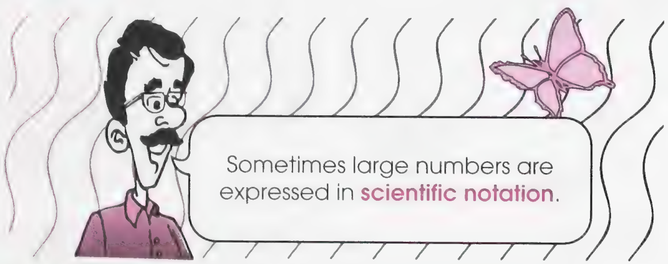
5. The Milky Way, of which the Earth's solar system is a part, is estimated to have 10^{11} stars. Write the number of stars in standard form.

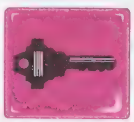


6. The world ant population is about 1 000 000 000 000 000. Write the number of ants as a power of ten.

7. A glass of water contains about 10^{25} molecules of water. Write the number of molecules in standard form.

Check your answers by turning to the Appendix.





Scientific notation is a way of expressing a number as the product of a power of ten and a number between 1 and 10.

Example 1

There are about 3.2×10^7 s in a year. Write the number of seconds in standard form.

Solution

Step 1: Write the power of ten as a whole number.

$$10^7 = 10\ 000\ 000$$

Step 2: Multiply 3.2 and 10^7.

$$3.2 \times 10^7 = 3.2 \times 10\ 000\ 000$$
$$= 32\ 000\ 000$$

There are about 32 000 000 s in a year.

Note: Multiplying a number by 10^7 moves the decimal place 7 places to the right.

32 000 000

8. Write each of the following numbers in standard form.

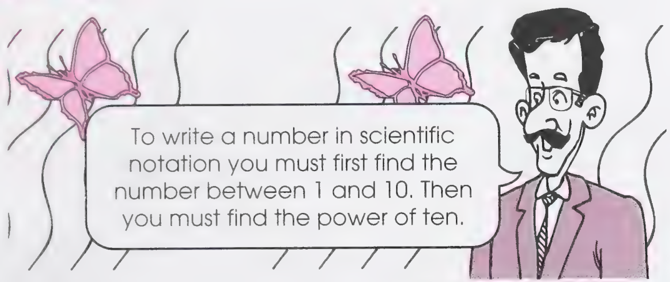
   a. The temperature at the centre of the Sun is about $2.0 \times 10^7\,°\mathrm{C}$.

b. One light-year is the distance travelled by light in one year. It is about 9.46×10^{12} km.

c. Sirius A, also called the Dog Star, is the brightest star visible in the sky. It has a mass of 4.65×10^{31} kg.

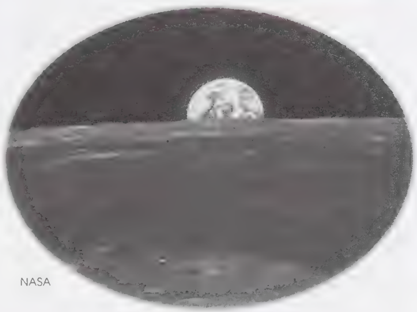
d. There are 1.2×10^{11} galaxies in the universe.

Check your answers by turning to the Appendix.



Example 2

The average distance from the surface of Earth to the surface of the Moon is about 376 000 km. Write this distance in scientific notation.


NASA

Solution

Step 1: Write the number 376 000 and show its decimal point. Then use a caret to locate the decimal point for the number between 1 and 10.

3‸76 000.

The number between 1 and 10 is 3.76.

Step 2: Find the exponent of the power of ten.

$3_{\wedge}\underbrace{76\,000}_{\text{5 digits}}.$

Count the number of places between the decimal point of 376 000. and the caret.

The exponent of the power of ten is 5.

Step 3: Write the number in scientific notation.

$$376\,000 = 3.76 \times 10^5$$

The average distance from the surface of Earth to the surface of the Moon is 3.76×10^5 km.

9. Write each of the following measurements in scientific notation.

a. The nearest star beyond the Sun is the very faint Proxima Centauri, which is about 40 300 000 000 000 km from Earth.

b. The largest telescope is the Hubble Space Telescope. It has a mass of over 11 000 kg.

c. The Hubble Space Telescope cost about \$155 000 000 000.

d. It takes light from the Great Galaxy in Andromeda, the most distant object that can be seen without a telescope, about 70 000 000 000 000 s to reach Earth.

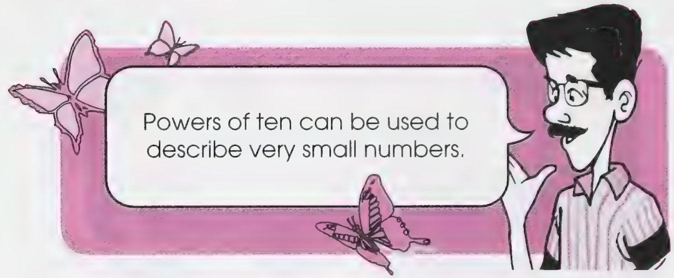
NASA

Check your answers by turning to the Appendix.

Use the Internet to discover more about space. You may find the Views of the Solar System site interesting; it is rated in the top 5% of all web sites. This is its uniform resource locator (URL):

http://bang.lanl.gov/solarsys/

Small Numbers



10. Each of the numbers in the following sequence is ten times as large as the number before it.

▢, ▢, ▢, 1, 10, 100, 1000,…

Use patterns to find the three numbers before 1.

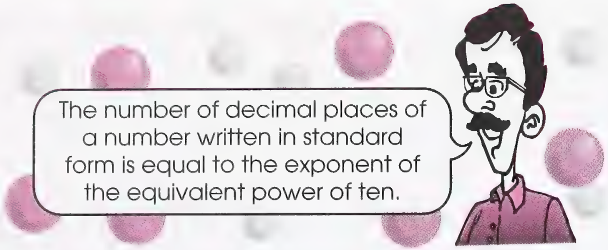
11. The sequence in question 10 can be written as powers of ten.

▢, ▢, ▢, 10^0, 10^1, 10^2, 10^3,…

Use patterns to find the three powers before 10^0.

12. Examine the corresponding forms of the numbers in the sequences in questions 10 and 11. What do you notice about each decimal number and its corresponding power of ten?

Check your answers by turning to the Appendix.



Power of Ten	Standard Form
10^{-1}	0.1
10^{-2}	0.01
10^{-3}	0.001
10^{-4}	0.0001
10^{-5}	0.000 01

13. Write each of the following measurements in standard form.

a. The diameter of a white blood cell is 10^{-3} cm.

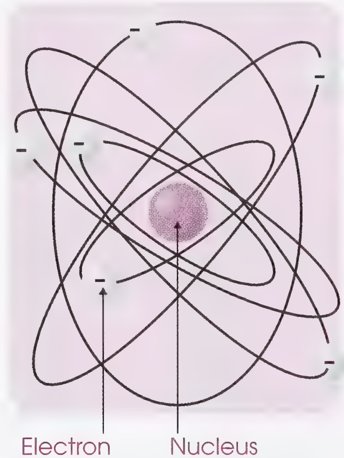
b. A type of bacteria known as Mycoplasma has a diameter of 10^{-5} cm and a mass of 10^{-16} g.

14. Write each of the following measurements as a power of ten.

a. The diameter of a carbon atom is 0.000 000 01 cm.

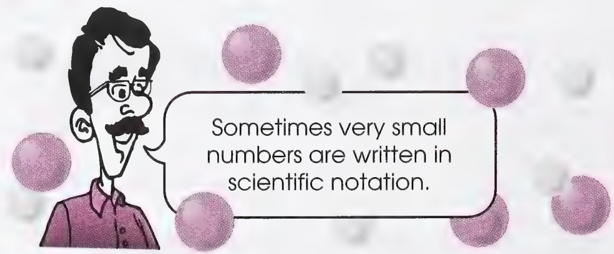
b. The diameter of the nucleus of a carbon atom is 0.000 000 000 001 cm.

c. An electron's diameter is 0.000 000 000 000 000 1 cm.



Note: This diagram is not drawn to scale.

Check your answers by turning to the Appendix.



Example 1

The potato spindle tuber virus has a diameter of 2.5×10^{-6} cm. Write this measurement in standard form.

NASA

Solution

Step 1: Change the power of ten to its standard form.

$$10^{-6} = \underbrace{0.000\,001}_{\text{6 places}}$$

Step 2: Multiply 2.5 and 10^{-6}.

$$\begin{aligned} 2.5 \times 10^{-6} &= 2.5 \times 0.000\,001 \\ &= 0.000\,002\,5 \end{aligned}$$

$$\begin{array}{rl} & 1 \text{ place} \\ + & 6 \text{ places} \\ \hline & 7 \text{ places} \end{array}$$

The diameter of the virus is 0.000 002 5 cm.

Note: Multiplying a number by 10^{-6} moves the decimal place 6 places to the left.

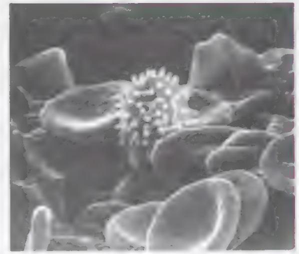
0.000 002 5

15. Write the following measurements in standard form.

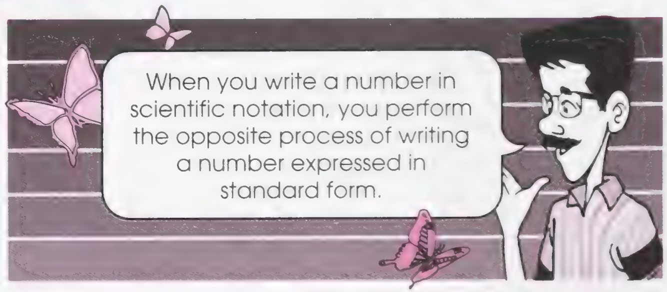
   a. The wavelength of yellow light is 5.8×10^{-5} m.

   b. The wavelength of one type of X-ray is about 1.28×10^{-8} m.

   c. A red blood cell has a length of 7.5×10^{-3} mm.


[1]

Check your answers by turning to the Appendix.



[1] Reproduced by permission from <http://ike.engr.washington.edu/software/eduimg/em/img0032.jpg> (7 July 1997). Copyright 1997 by International Business Machines Corporation.

Example 2

The diameter of a hydrogen atom is about 0.000 08 mm. Write this measurement in scientific notation.

Solution

Step 1: Write the number 0.000 08. Use a caret to locate the decimal point for a number between 1 and 10.

0.000 08^

The number between 1 and 10 is 8.

Step 2: Find the exponent of the power of ten. Remember it will be a negative number.

Count the number of places between the decimal point of 0.000 08 and the caret.

$\underbrace{0.000\,08}_{\text{5 places}}$^

The exponent of the power of ten is -5.

Step 3: Write the number in scientific notation.

$$0.000\,08 = 8 \times 10^{-5}$$

The diameter of a hydrogen atom is about 8×10^{-5} mm.

16. Write each of the following numbers in scientific notation.

a. About 0.000 018 of the dry air at sea level is neon.

b. About 0.0003 of all water on Earth evaporates each year.

Check your answers by turning to the Appendix.

Did You Know?

Electron microscopes enlarge objects by a greater scale factor than optical or light microscopes. What role did Canadians have in the development of the electron microscope?

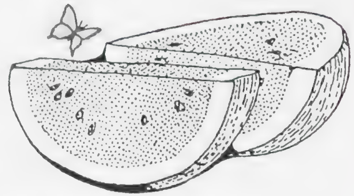
Find and read the article "The First Practical Electron Microscope" in the Appendix.

Use the Internet to discover more about the scanning electron microscope (SEM). This is the uniform resource locator (URL) of a site that you may find interesting; it has a picture library of SEM images (insects, plants, food, human pictures, and so on).

http://surf.eng.iastate.edu/%7Ekarie/SEM.html

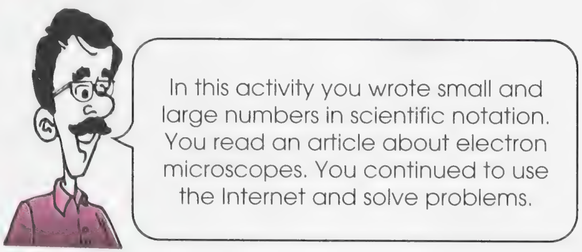
Now Try This

Use a problem-solving strategy to answer the following question.



17. On Monday morning, a store put out some watermelons, and six were sold. On Tuesday morning, the number of leftover watermelons was doubled, and six were sold. On Wednesday morning, the number of leftover watermelons was tripled, and six were sold. If there were no watermelons left on Wednesday afternoon, how many watermelons were put out on Monday morning?

Check your answer by turning to the Appendix.

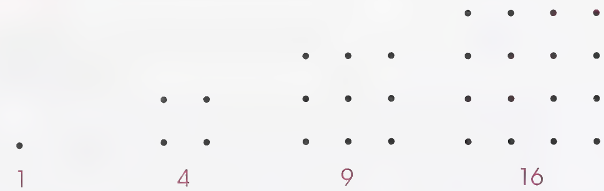


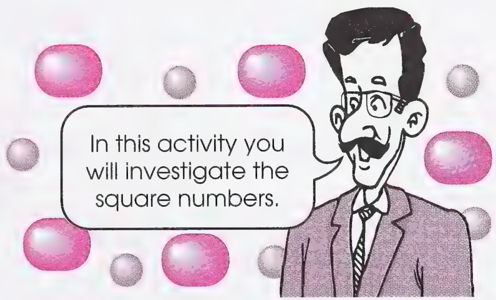
Activity 2: Squares and Square Roots

The ancient Greek mathematicians spent considerable time in arranging dots into geometric shapes and then counting the dots.

They discovered that some numbers such as 1, 3, 6, and 10 are **triangular numbers**.

Some other numbers such as 1, 4, 9, and 16 are **square numbers**.





Following is a sequence of the first ten square numbers.

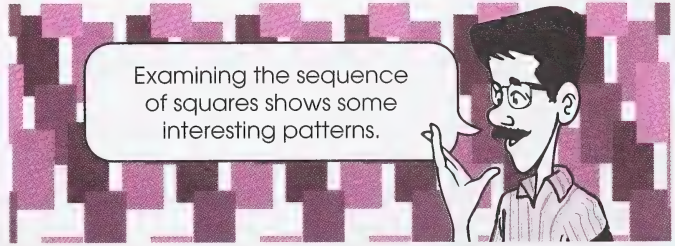
1, 4, 9, 16, 25, 36, 49, 64, 81, 100,…

The sequence of square numbers was formed by multiplying the consecutive counting numbers by themselves.

$$\underset{1\times1}{1},\ \underset{2\times2}{4},\ \underset{3\times3}{9},\ \underset{4\times4}{16},\ \underset{5\times5}{25},\ \underset{6\times6}{36},\ \underset{7\times7}{49},\ \underset{8\times8}{64},\ \underset{9\times9}{81},\ \underset{10\times10}{100},\ldots$$

The sequence of square numbers can be written using powers of 2.
Note: A power of 2 is a number multiplied by itself.

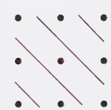
$1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2, 8^2, 9^2, 10^2, \ldots$



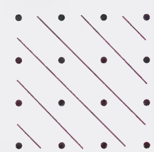
1. Look at the sequence of square numbers when it is arranged like this:

$1^2 = 1$

$2^2 = 1+2+1$



$3^2 = 1+2+3+2+1$



$4^2 = 1+2+3+4+3+2+1$

a. Make a similar diagram for 5^2 and write a number sentence to describe the square.

b. Make a similar diagram for 6^2 and write a number sentence to describe the square.

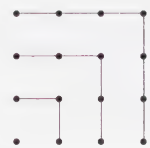
c. Use this pattern to write a number sentence to describe 10^2.

2. Look at the sequence of square numbers when it is arranged like this:

$1^2 = 1$

$2^2 = 1 + 3$

$3^2 = 1 + 3 + 5$



$4^2 = 1 + 3 + 5 + 7$

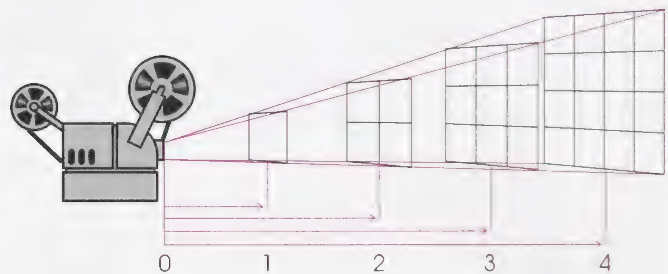
a. Make a similar diagram for 5^2 and write a number sentence to describe the square.

b. Make a similar diagram for 6^2 and write a number sentence to describe the square.

c. Write a number sentence to describe 10^2. **Note:** Do **not** make a diagram.

3. Walter was watching home movies with his family. He noticed that the area of the picture on the screen was related to the distance the projector was from the screen.

This relationship is illustrated in the following diagram.



a. Use the diagram to complete a table like this.

Distance from Screen (units)	Area of Picture (square units)
1	
2	
3	
4	

b. Write a formula to describe how the area of the picture on the screen is related to the distance of the projector from the screen. Let a be the area (in square units) of the picture. Let d be the distance (in units) from the screen.

c. Use the formula to calculate the area of the picture on the screen if the projector is 5 units from the screen.

4. The distance an object will fall when it is dropped is given by the following formula, where d is the distance (in metres) and t is the time (in seconds).

$$d = 5t^2$$

An object is dropped from the top of the CN Tower. How far will the object fall in 4 s?

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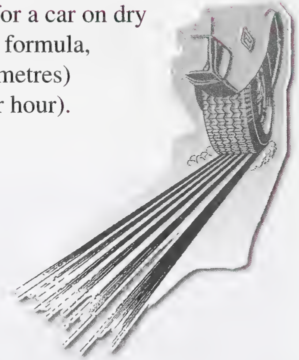
5. The approximate braking distance for a car on dry pavement is given by the following formula, where d is the braking distance (in metres) and s is the speed (in kilometres per hour).

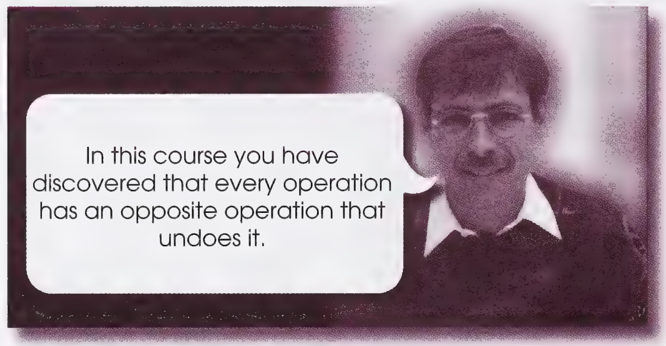


$$d = \frac{s^2}{210}$$

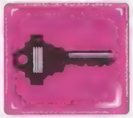
a. What is the braking distance of a vehicle travelling at 50 km/h?

b. What is the braking distance of a vehicle travelling at 100 km/h?

Check your answers by turning to the Appendix.



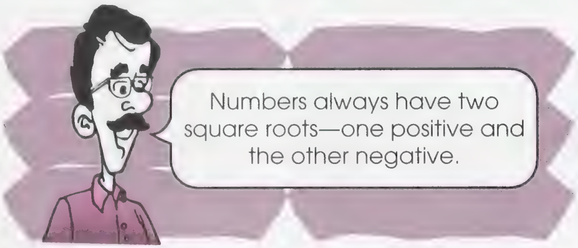
If subtraction is the opposite of addition and division is the opposite of multiplication, what is the opposite of squaring a number? To undo the operation of squaring, find the **square root**.



A square root is a number, which when multiplied by itself, results in the given number.

For example, $2^2 = 2 \times 2 = 4$ and $(-2)^2 = (-2) \times (-2) = 4$.

Therefore, the square roots of 4 are 2 and -2.



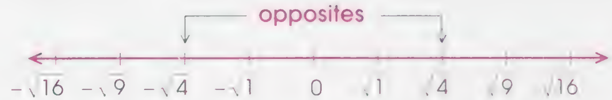
The symbol $\sqrt{4}$ stands for the **positive** square root of 4. The positive square root may also be called the **principal** square root.

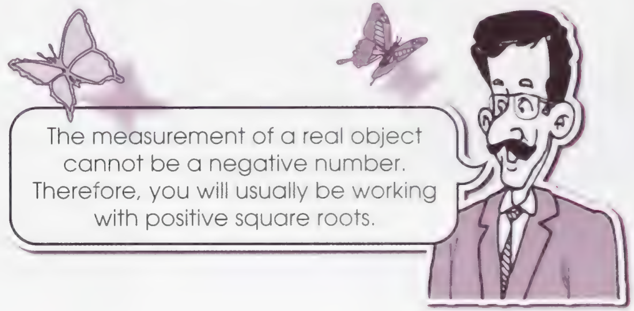
$$\sqrt{4} = 2$$

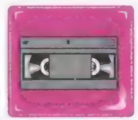
The symbol $-\sqrt{4}$ stands for the **negative** square root of 4.

$$-\sqrt{4} = -2$$

As you can see from the following number line, $\sqrt{4}$ and $-\sqrt{4}$ are opposites.



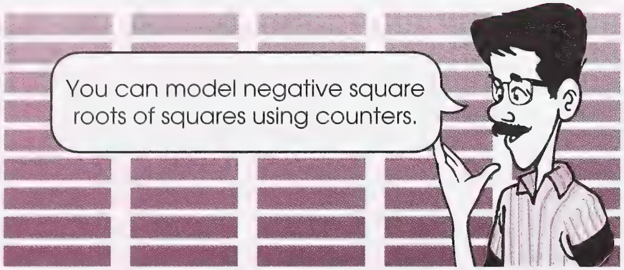




Gather small cubes and the video program *Number Theory* from the series *Math Moves* to discover how you can model positive square roots.

6. View the final segment entitled "Square Root" and do the video assignment.

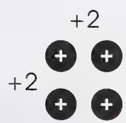
Check your answers by viewing the video.



Stephen used counters to model the positive square root and the negative square root of 4.

To find the positive square root of 4, Stephen asked himself this question, "What positive number multiplied by itself is 4?" The answer is $+2$.

To model the positive square root of 4, Stephen arranged four positive counters in a square array and showed the factors.



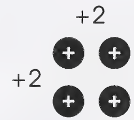
$(+2)\times(+2)=+4$

When multiplied by itself, $+2$ results in 4.

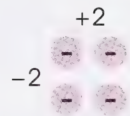
$\therefore\ \sqrt{4}=2$

To model the negative square root of 4, Stephen asked himself this question, "What negative number multiplied by itself is 4?" The answer is -2.

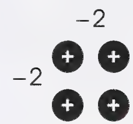
To model the negative square root of 4, Stephen used this reasoning. For each negative factor, counters are exchanged for their opposites once. Therefore, Stephen first arranged four positive counters in a square array.



To show $(-2)\times(+2)$, Stephen exchanged the counters for their opposites.



To show $(-2)\times(-2)$, Stephen exchanged the counters for their opposites.



$(-2)\times(-2)=+4$

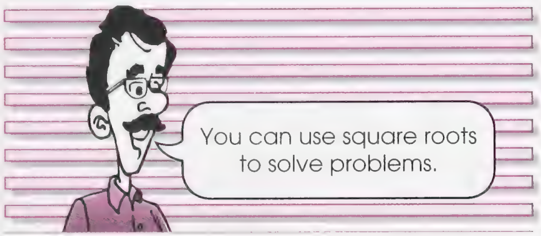
When multiplied by itself, -2 results in 4.

$\therefore\ -\sqrt{4}=-2$

To answer question 7, you will need the counters from the Appendix of Module 1.

7. a. Model the positive and negative square roots of 9.
   b. Model the positive and negative square roots of 16.

Check your answers by turning to the Appendix.



Example 1

The area of Esther's square flower bed is $25\ \text{m}^2$. Find the length of one side of the flower bed.

Solution

Step 1: Write the formula for the area of a square and substitute the given area.

$$A = s^2$$
$$25 = s^2$$

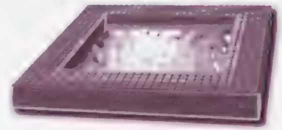
Step 2: To solve for s, ask yourself, "What is the square root of 25, or what number multiplied by itself is 25?" Since you need a measurement, give only the positive square root of 25.

$$25 = s^2$$
$$5 = s$$

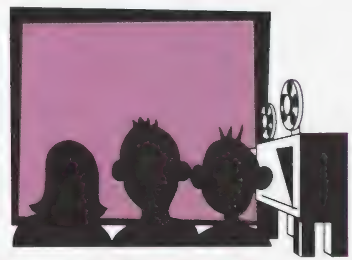
$$5 \times 5 = 25$$

The length of one side of the flower garden is 5 m.

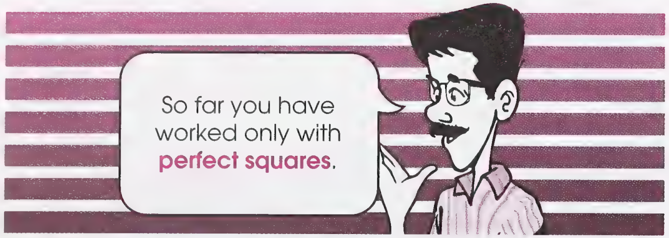
8. If a square pool has a surface area of $64\ \text{m}^2$, what is the length of one side of the pool?

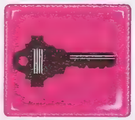


9. If the area of a picture on a screen is 36 square units, how far is the projector from the screen? **Hint:** See question 3 in this activity.



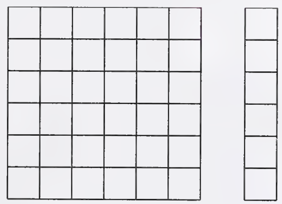
Check your answers by turning to the Appendix.



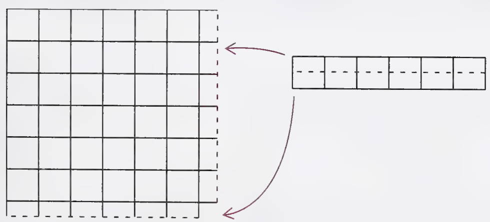


A perfect square is a number that has a **counting number** as its square root. The counting numbers are 1, 2, 3, 4, 5, 6,… Counting numbers are also called **natural numbers**.

10. Hannah used grid paper to show that the square root of 42 is not a counting number. She cut out 42 squares from grid paper and tried to arrange them in a square. The largest square she could make was a 6×6 square.



She then cut the 6 leftover squares in half lengthwise and placed them as shown in the following diagram.



Use the diagram to estimate the square root of 42.

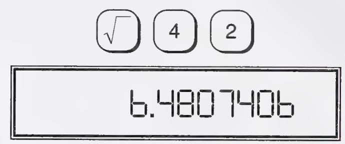
11. **a.** Use Hannah's method (see question 10) to show that $\sqrt{56}$ is not a counting number.

b. Use your diagram to estimate $\sqrt{56}$.

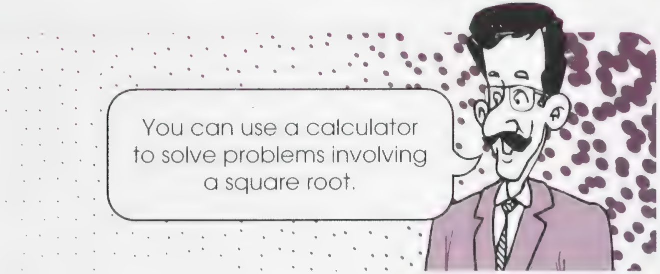
Check your answers by turning to the Appendix.

You can approximate the square root of non-perfect squares using a calculator.

For example, to find the square root of 42 press these keys:



The square root of 42 is about 6.5; this is an approximation.



Example 2

A player at third base must throw the ball to the catcher at home plate.

If the area of a baseball diamond is about $751\ \text{m}^2$, what is the distance from third base to home plate? **Hint:** To calculate the area of a square, use the following formula, where A is the area in square units and s is the length of each side in linear units.

$$A = s^2$$

Solution

Step 1: Write the formula for the area of a square and substitute the given area.

$$A = s^2$$

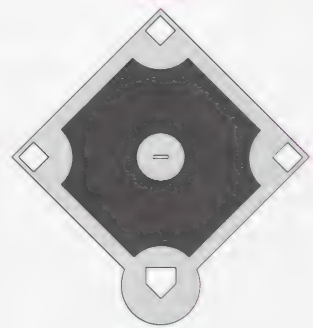
$$751 = s^2$$

Step 2: To solve for s, ask yourself, "What number multiplied by itself is 751?" Since you require a measurement, give only the positive square root. **Hint:** Use a calculator.

$$\sqrt{751} = s$$

$$27.4 \doteq s$$

The distance from third base to home plate is about 27.4 m.



Example 3

There was a car accident. The police who arrived at the scene measured the skid marks and found that the braking distance was 62 m. At what speed was the car travelling? **Hint:** To calculate the speed the car was travelling, use the following formula, where d is the braking distance (in metres) and s is the speed (in km/h).

$$d = \frac{s^2}{210}$$

Solution

Step 1: Write the formula for the braking distance and substitute the given distance.

$$d = \frac{s^2}{210}$$

$$62 = \frac{s^2}{210}$$

Step 2: Multiply each side of the equation by 210 to isolate the variable.

$$210(62) = \cancel{210}\left(\frac{s^2}{\cancel{210}}\right)$$

$$13\,020 = s^2$$

Step 3: To solve for s, ask yourself, "What is the square root of 13 020?" Since a measurement is required, use only a positive square root. **Hint:** Use a calculator.

$$\sqrt{13\,020} = s$$

$$114 \doteq s$$

The car was travelling at a speed of about 114 km/h.

12. The area of a certain square field is $3000\ \text{m}^2$. What is the length of one side of the field?

13. Jason dropped a penny from a bridge into the water, a distance of 50 m. How long did it take the penny to hit the water?

Hint: To calculate how long it took the penny to hit the water, use the following formula, where d is the distance (in metres) and t is the time (in seconds).

$$d = 5t^2$$

Check your answers by turning to the Appendix.

You may wish to discover more about squares and square roots or other math topics on the Internet by visiting a general mathematics site such as Middle School Student Centre. This is the uniform resource locator (URL) of their annotated home page:

http://forum.swarthmore.edu/students/

Click on Ask Dr. Math. Then select "Middle School."

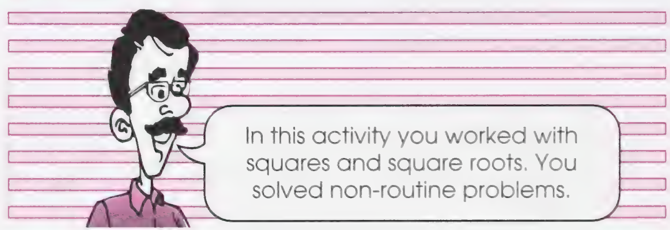
Now Try This

Use a problem-solving strategy to answer each of the following questions.

14. Mrs. Mahr is nine times the age of her daughter. In three years Mrs. Mahr will be five times as old as her daughter. What are their ages now?

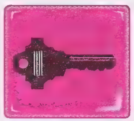
15. Jonathon has 20 more nickels than dimes in his piggy bank. If the total value of the nickels and dimes is \$8.50, how many coins of each type does he have?

Check your answers by turning to the Appendix.



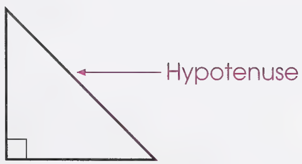
Activity 3: The Pythagorean Relation

In the sixth century B.C., a mathematician and philosopher known as Pythagoras (approximately 575 B.C.–495 B.C.) identified a numerical relationship between the lengths of the sides of a **right triangle**.

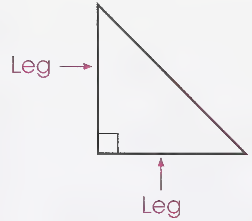


A right triangle has an angle with a measure of 90°.

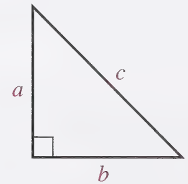
The side opposite the right angle is the **hypotenuse**. It is the longest side.



The other two sides of the triangle are the **legs**.



Each side of a right triangle is often represented by a letter.



You will now investigate the relationship Pythagoras identified. For this investigation, locate Pythagorean Puzzle 1 and Pythagorean Puzzle 2 in the Appendix.

Notice how squares have been constructed on the legs and hypotenuse of each right triangle. In each triangle, Square A has a side of a; therefore, the area of Square A is a^2. Square B has a side of b; therefore, the area of Square B is b^2. Square C has a side of c; therefore, the area of Square C is c^2.

1. Photocopy Pythagorean Puzzle 1 and glue the photocopy to heavy paper to make it easier to handle.

   Cut out Square A and Square B. Then cut Square A along the dotted lines.

   a. Arrange Square B and the four pieces of Square A over Square C. Do Square B and the four pieces of Square A cover Square C?

   b. What conclusion can you make about the area of Square C? Write the relationship as an equation.

2. Photocopy Pythagorean Puzzle 2 and glue the photocopy to heavy paper to make it easier to handle.

   Cut out Square A and Square B. Then cut Square A along the dotted lines.

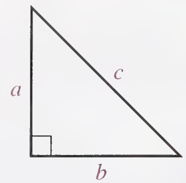
   a. Arrange Square B and the four pieces of Square A over Square C. Do Square B and the four pieces of Square A cover Square C?

   b. What conclusion can you make about the area of Square C? Write the relationship as an equation.

Check your answers by turning to the Appendix.

The Pythagorean relation states that in a right triangle, the sum of the squares of the legs is equal to the square of the hypotenuse.

The following diagram shows the Pythagorean relation.

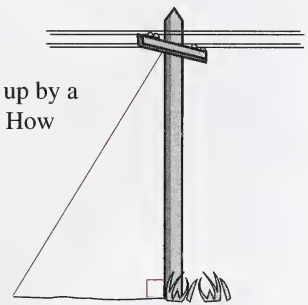


$$a^2 + b^2 = c^2$$

The Pythagorean relation can be used to solve many problems involving right triangles.

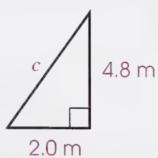
Example 1

A power pole that is 4.8 m high is held up by a guy wire anchored 2.0 m from its base. How long is the guy wire?



Solution

Step 1: Draw and label a diagram to represent the problem.



Step 2: Write the formula for the Pythagorean relation and substitute the given values. Then simplify the equation.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 4.8^2 + 2.0^2 &= c^2 \\ 23.04 + 4.0 &= c^2 \\ 27.04 &= c^2 \end{aligned}$$

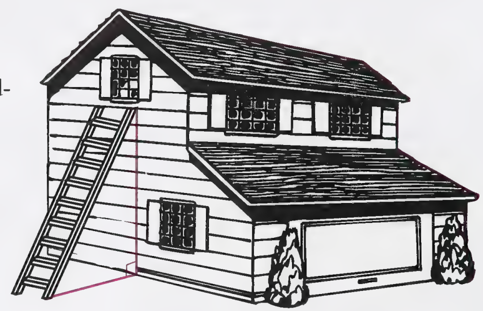
Step 3: To solve for c, ask yourself, "What is the square root of 27.04?" Since a measurement is required, use only a positive square root. **Hint:** Use a calculator.

$$\begin{aligned} \sqrt{27.04} &= c \\ 5.2 &= c \end{aligned}$$

The guy wire is 5.2 m long.

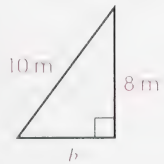
Example 2

A ladder that is 10 m long reaches a second-storey window. If the distance from the window sill to the ground is 8 m, how far is the foot of the ladder from the house?



Solution

Step 1: Draw and label a diagram to represent the problem.



Step 2: Write the formula for the Pythagorean relation and substitute the given values. Then simplify the equation.

$$a^2 + b^2 = c^2$$
$$8^2 + b^2 = 10^2$$
$$64 + b^2 = 100$$

Step 3: To isolate the variable, add -64 to each side of the equation.

$$\begin{array}{rcr} \cancel{64} + b^2 & = & 100 \\ \cancel{-64} & & -64 \\ \hline b^2 & = & 36 \end{array}$$

Step 4: To solve for b, ask yourself, "What is the square root of 36?"

$b = 6$

Since a measurement is required, use only a positive square root.

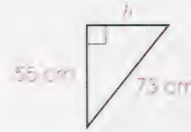
The foot of the ladder is 6 m from the house.

Use a calculator to answer questions 3 to 8.

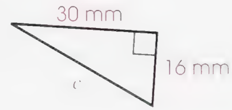
3. For each of the following triangles, calculate the length of the missing side.

a.

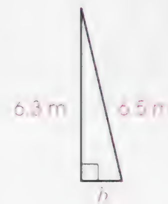
b.



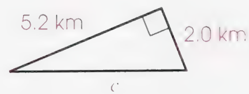
c.



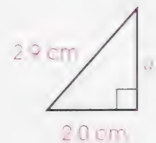
d.



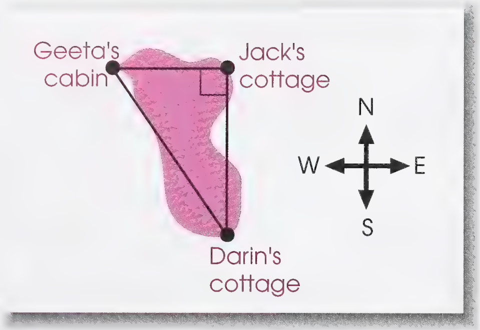
e.



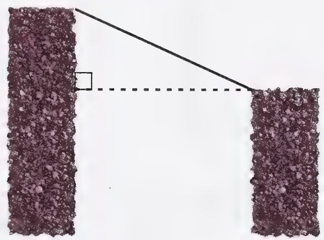
f.



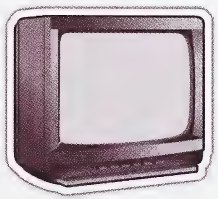
4. Darin's cottage is 2.4 km south of Jack's cottage. Geeta's cabin is 3.0 km northwest of Darin's cottage. Geeta's cabin is west of Jack's cottage. How far is it across the lake from Geeta's cabin to Jack's cottage?



5. Two walls are connected by a steel cable that is 13 m long. If one wall is 11 m high and the other is 6 m high, find the distance between the walls.



6. A television screen is a rectangle that measures 30 cm wide and 22 cm high. What is the diagonal measure of the screen?



7. A baseball diamond is a square with sides of about 27.4 m. A player at second base must throw a ball to home plate. How far must the player throw the ball?

8. A box is 120 cm long and 25 cm wide. What is the length of the longest ski pole that could be packed to lie flat in the box?

Check your answers by turning to the Appendix.

Did You Know?

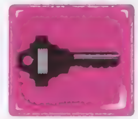
Houses would look very strange if there were no way of making 90° angles.

Fortunately there are many tools to help builders make square corners.

The ancient Egyptians did not have any of these tools. How do you think they made 90° angles?

To make a square corner, the Egyptians tied knots to divide a rope into 12 equal parts. The rope was then shaped into a triangle with lengths of 3, 4, and 5 units. Stakes were used to hold the triangle securely.

Why did this method of making 90° angles work? The numbers 3, 4, and 5 make up a **Pythagorean triple** and a triangle with sides of 3, 4, and 5 units is a right triangle.



Three numbers form a Pythagorean triple if the sum of the squares of the two smaller numbers equals the square of the third number.

9. Is each of the following a Pythagorean triple? Answer **yes** or **no**.

a. 5, 7, 10 **b.** 7, 24, 25 **c.** 9, 13, 14
d. 5, 12, 13 **e.** 7, 20, 21 **f.** 1, 10, 11

10. One Pythagorean triple is 3, 4, and 5.

a. Is 6, 8, and 10 a Pythagorean triple?

b. Is 9, 12, and 15 a Pythagorean triple?

c. Name another Pythagorean triple that is a multiple of 3, 4, and 5.

11. What seems to be a general rule for finding more Pythagorean triples once you know one?

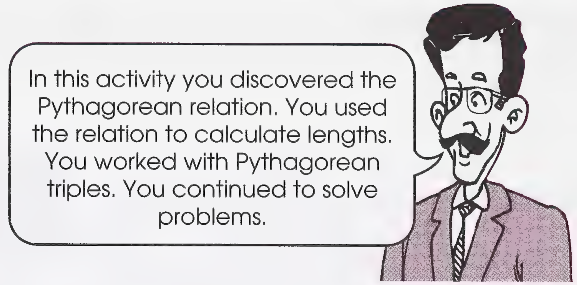
Check your answers by turning to the Appendix.

Now Try This

Use a problem-solving strategy to answer the following question.

12. In a group of students, 25 students brought an apple in their lunch, 20 students brought a cookie in their lunch, 14 students brought both an apple and a cookie, and 3 students brought neither an apple nor a cookie. How many students were there altogether?

Check your answer by turning to the Appendix.



Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

In this activity you wrote numbers in scientific notation.

To express a number in scientific notation, write it as a product of two factors. Use boxes as an aid to recall the two factors.

First Factor	a number between 1 and 10

Second Factor	a power of ten

Example 1

Write 3 480 000 in scientific notation.

Solution

First Factor	a number between 1 and 10	3.48	$3_{\wedge}480\,000.$
Second Factor	a power of ten	10^{6}	$3_{\wedge}\underbrace{480\,000.}_{6 \text{ places}}$

$\therefore\ 3\,480\,000 = 3.48 \times 10^{6}$

1. Is each of the following numbers in scientific notation?

   a. 2.39×10^{5} **b.** 0.13×10^{8} **c.** 8.4×10^{2}

2. Express each of the following numbers in scientific notation.

   a. 893 **b.** 8 930 000 **c.** 8930

Check your answers by turning to the Appendix.

You can use a similar method to express a very small number in scientific notation.

Example 2

Write 0.000 000 014 8 in scientific notation.

Solution

First Factor	a number between 1 and 10	1.48	$0.000\,000\,01_{\wedge}4\,8$
Second Factor	a power of ten	10^{-8}	$\underbrace{0.000\,000\,01}_{8 \text{ places}}{}_{\wedge}4\,8$

$\therefore\ 0.000\,000\,014\,8 = 1.48 \times 10^{-8}$

3. Is each of the following numbers in scientific notation?

   a. 125×10^{-4} **b.** 12.3×10^{-6} **c.** 0.64×10^{-7}

4. Express each of the following numbers in scientific notation.

   a. 0.008 93 **b.** 0.000 893 **c.** 0.000 089 3

5. Locate the puzzle "Why Couldn't Orgo Keep His Waterbed a Secret?" in the Appendix. Photocopy or pull out the page and complete the puzzle.

Check your answers by turning to the Appendix.

Enrichment

What is the name of the largest number that you can think of? Is it a million? a billion? a trillion?

In the comic strip, Schroeder thought of a **googol**. A googol is the name for 1 followed by 100 zeros.

The largest number that has a name is a **googolplex**. A googolplex is the name for 1 followed by a googol zeros.

Here is a list of the names and power forms of some other large numbers.

Name	Power Form
million	10^6
billion	10^9
trillion	10^{12}
quadrillion	10^{15}
quintillion	10^{18}
sextillion	10^{21}
septillion	10^{24}
octillion	10^{27}
nonillion	10^{30}
decillion	10^{33}

1. A zillion is an informal name for an extremely large number. How do you suppose the name "zillion" may have originated?

2. Write each of the following numbers as a number in standard form and as a power of ten.

   a. ten million
   b. one hundred billion
   c. one hundred octillion
   d. ten trillion

3. It is estimated that the insect population of the world is a quintillion (10^{18} or 1 000 000 000 000 000 000). Shawn killed a fly and said, "Now there are 999 999 999 999 999 999 insects left." Was Shawn's statement correct? Why or why not?

4. On July 8, 1997, Canada's national debt was given as $596 147 559 260.89.

   a. Round this number to the nearest billion.
   b. Write the number you wrote in 5.a. in words.
   c. Write the number you wrote in 5.a. in scientific notation.

Check your answers by turning to the Appendix.

Use the Internet to find out what Canada's national debt is today. How does it compare to the national debt of the United States? You may find the following site helpful.

http://www.cam.org/˜mdavies/cgi-bin/CanClock.cgi

Conclusion

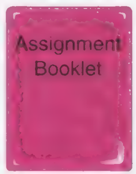
In this section you explored powers and square roots. You wrote numbers in scientific notation. You investigated the relationship between squares and square roots. You found the square root of numbers. You discovered the relation, called the Pythagorean relation, between the sides of a right triangle.

You discovered that scientific notation can be used to describe very large numbers and very small numbers.

Did you know monarch butterflies can fly great distances? A tagged female butterfly released in Canada was recaptured in Mexico four months later; it had travelled 3.43×10^{6} m.

Did you know that a midge has the fastest wing-beat of any insect? Its wing-beat under normal conditions is 6.28×10^{4} beats per minute. The muscular contraction-expansion cycle further represents the fastest muscle movement ever measured. Each cycle is 4.5×10^{-4} s.

Assignment



You are now ready to complete the assignment for Section 2.

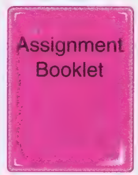
Module Summary

In this module you worked with proportional situations, powers, and square roots.

Numbers are used to describe and measure many activities in the world, including gardening and landscaping. It is important that you become comfortable dealing with numbers and their applications.

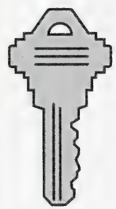
Having completed this module, you know that a box of bone-meal fertilizer labelled 2-11-0 means that it contains 2% nitrogen, 11% phosphorus, and 0% potassium. You can also appreciate the enormity of the following fact. During the last decade more than 1.43×10^8 t of nitrogen, phosphorus, and potassium were used in fertilizers throughout the world.

Final Module Assignment



You are now ready to complete the final module assignment.

APPENDIX



- Glossary
- Suggested Answers
- Articles/Cartoons
- Puzzle
- Cut-out Learning Aids

Glossary

Commission: the pay an employee earns based on a percentage of the sales made by the employee

Compound interest: the interest earned (charged) on an amount of money and added to the principal to earn (charge) more interest in the following year

Dilatation: a kind of transformation in which the image is an enlargement or a reduction of the original

Discount: the amount by which the regular price is reduced

Interest: the amount paid for the use of money

Percent: a ratio having 100 as its second term

Perfect square: a number that has a counting number (natural number) as its square root

Power of ten: power with a base of ten

Principal: money which is invested or borrowed

Problem: a task for which the method of finding the answer (as well as the answer) is not immediately known

Proportions: equations that show two ratios are equivalent

Pythagorean relation: the relationship that shows that in a right triangle, the sum of the squares of the legs is equal to the square of the hypotenuse; $a^2 + b^2 = c^2$

Pythagorean triple: three numbers whose sum of the squares of the two smaller numbers equals the square of the third number

Rate: a comparison of quantities measured in different units

Ratio: a comparison of quantities measured in the same unit

Right triangle: a triangle with an angle of 90°

Scale drawing: a drawing used to accurately picture a person, animal, or thing that is too large or too small to be drawn actual size

Scale factor: in a drawing, the ratio of the drawn length to the actual length expressed as a single number

Scale model: a likeness of an object that has the same proportions as the object

Scientific notation: a way of expressing a number as the product of a power of ten and a number between 1 and 10

Square root: a number which, when multiplied by itself, results in the given number

Standard form: the usual form of a number

Technology: the application of tools, materials, and processes to problem solving; more specifically, devices and systems used in processing, transferring, storing, and communicating information through electronic media

Term: the numbers in a ratio

The first number in a ratio is called the first term; the other numbers in order, are called the second term, the third term, and so on.

Suggested Answers

Section 1: Activity 1

1. a. gold:silver:bronze

$$\begin{array}{ccccc} & 5 & : & 5 & : & 5 \\ & \downarrow \div 5 & & \downarrow \div 5 & & \downarrow \div 5 \\ = & 1 & : & 1 & : & 1 \end{array}$$

The ratio of gold to silver to bronze medals won by Switzerland in the 1988 Winter Olympics is 1 to 1 to 1.

b. **Step 1:** Find the total number of medals won by Switzerland in the 1988 Winter Olympics.

$$5+5+5=15$$

Switzerland won 15 medals.

Step 2: Find the ratio of gold medals to total medals won by Switzerland in the 1988 Winter Olympics.

gold:total

$$\begin{array}{cccc} & 5 & : & 15 \\ & \downarrow \div 5 & & \downarrow \div 5 \\ = & 1 & : & 3 \end{array}$$

The ratio of gold medals to the total medals won by Switzerland in the 1988 Winter Olympics is 1 to 3.

2. absorbed:inhaled

$$\begin{array}{cccc} & 0.57 & : & 11.4 \\ & \downarrow \times 100 & & \downarrow \times 100 \\ = & 57 & : & 1140 \\ & \downarrow \div 57 & & \downarrow \div 57 \\ = & 1 & : & 20 \end{array}$$

The ratio of oxygen absorbed to air inhaled is 1 to 20.

3. a. red:yellow:black

$$\begin{array}{cccccc} & 2 & : & 1.25 & : & 0.25 \\ & \downarrow \times 100 & & \downarrow \times 100 & & \downarrow \times 100 \\ = & 200 & : & 125 & : & 25 \\ & \downarrow \div 25 & & \downarrow \div 25 & & \downarrow \div 25 \\ = & 8 & : & 5 & : & 1 \end{array}$$

The ratio of red to yellow to black paint is 8 to 5 to 1.

b. **Step 1:** Find the number of parts of brown paint.

$$8+5+1=14$$

There are 14 parts of brown paint.

Step 2: Find the ratio of red to yellow to black to brown paint.

red:yellow:black:brown   8:5:1:14

The ratio of red to yellow to black to brown paint is 8 to 5 to 1 to 14.

4. Great Star of Africa:Cullinan

$$
\begin{aligned}
& 530.2 : 3106 \\
& \downarrow \times 10 \quad \downarrow \times 10 \\
= \ & 5302 : 31\,060 \\
& \downarrow \div 2 \quad \downarrow \div 2 \\
= \ & 2651 : 15\,530
\end{aligned}
$$

The ratio of the mass of the Great Star of Africa to the Cullinan is 2651 to 15 530.

5. a.

Position of Gear	Number of Teeth on Front Gear	Number of Teeth on Back Gear	Gear Ratio
1st	52	28	1.857
2nd	52	24	2.167
3rd	52	20	2.600
4th	52	17	3.059
5th	52	14	3.714

b. The fifth gear has the greatest gear ratio. The first gear has the lowest gear ratio.

6. a. $\frac{\text{hits}}{\text{times at bat}}$ $\frac{22}{75} = 0.293$

Tara had a batting average of 0.293.

b. $\frac{\text{hits}}{\text{times at bat}}$ $\frac{15}{60} = 0.250$

Jocelyn had a batting average of 0.250.

c. Compare the batting averages.

$0.293 > 0.250$

Tara is the better hitter; she had the higher batting average.

7. Step 1: Write a proportion. Let ▢ be the number of red marbles and ▢ be the number of white marbles.

red:white:blue

$$
\begin{aligned}
& 2 : 3 : 4 \\
= \ & \square : \square : 12
\end{aligned}
$$

Step 2: Find the missing term. Use this reasoning. Because 4 was multiplied by 3 to get 12; multiply each of the other terms by 3.

red:white:blue

$$\begin{array}{ccccc} & 2 & : & 3 & : & 4 \\ & \downarrow \times 3 & & \downarrow \times 3 & & \downarrow \times 3 \\ = & 6 & : & 9 & : & 12 \end{array}$$

There are 6 red marbles and 9 white marbles.

8. **Step 1:** Write a proportion.

gold:total

$$\begin{array}{ll} & 14 : 24 \\ = & \square : 72 \end{array}$$

Step 2: Find the missing term. Use this reasoning. Because 24 was multiplied by 3 to get 72, multiply the other term by 3.

$$\begin{array}{cccc} & 14 & : & 24 \\ & \downarrow \times 3 & & \downarrow \times 3 \\ = & 42 & : & 72 \end{array}$$

There are 42 g of gold in the ring.

9. **Step 1:** Find the total number of parts.

$$37 + 3 = 40$$

Step 2: Write a proportion. Let $\square$ be the amount of silver and $\square$ be the amount of copper.

silver:copper:total

$$\begin{array}{ll} & 37 : 3 : 40 \\ = & \square : \square : 800 \end{array}$$

Step 3: Find the missing terms. Use this reasoning. Because 40 was multiplied by 20 to get 800, multiply each of the other terms by 20.

silver:copper:total

$$\begin{array}{cccccc} & 37 & : & 3 & : & 40 \\ & \downarrow \times 20 & & \downarrow \times 20 & & \downarrow \times 20 \\ = & 740 & : & 60 & : & 800 \end{array}$$

There are 740 g of silver and 60 g of copper in the goblet.

10. **a.**

gold:silver

$$\frac{1}{5} : \frac{4}{5}$$

$$5\left(\frac{1}{5}\right) : 5\left(\frac{4}{5}\right)$$

$$1 : 4$$

The ratio of gold to silver is 1 to 4.

b. Step 1: Write the ratio of the amount of gold to the total amount of precious metals.

$1 + 4 = 5$

gold:total $\quad 1:5$

Step 2: Write a proportion. Let the mass of the gold be $\square$.

gold:total

$$\begin{aligned} &1 : 5 \\ = &\square : 110 \end{aligned}$$

Step 3: Find the missing term. Use this reasoning. Because 5 was multiplied by 22, multiply the other term by 22.

gold:total

$$\begin{array}{ccc} 1 & : & 5 \\ \downarrow \times 22 & & \downarrow \times 22 \\ = \boxed{22} & : & 110 \end{array}$$

The mass of gold in the bulb is 22 g.

11. Step 1: Find the total number of parts.

$95 + 4 + 1 = 100$

Step 2: Write a proportion. Let $\square$ be the amount of copper, $\bigcirc$ be the amount of tin, and $\diamond$ be the amount of zinc.

copper:tin:zinc:total

$$\begin{aligned} &95 : 4 : 1 : 100 \\ = &\square : \bigcirc : \diamond : 4.5 \end{aligned}$$

Step 3: Find the missing terms. Use this reasoning. Because 100 was multiplied by 0.045 to 4.5, multiply each of the other terms by 0.045.

copper:tin:zinc:total

$$\begin{array}{ccccccc} 95 & : & 4 & : & 1 & : & 100 \\ \downarrow \times 0.045 & & \downarrow \times 0.045 & & \downarrow \times 0.045 & & \downarrow \times 0.045 \\ = \boxed{4.275} & : & 0.18 & : & 0.045 & : & 4.5 \end{array}$$

a. The mass of the copper is 4.275 g.
b. The mass of the tin is 0.18 g.
c. The mass of the zinc is 0.045 g.

12. Step 1: Find the total number of parts.

$2 + 3 = 5$

Step 2: Write a proportion. Let $\boxed{}$ be Meredith's share and $\bigcirc$ be Raschid's share.

Meredith's share:Raschid's share:total

$$\begin{aligned} &2 : 3 : 5 \\ = &\boxed{} : \bigcirc : 1500 \end{aligned}$$

Step 3: Find the missing terms. Use this reasoning. Because 5 was multiplied by 300 to get 1500, multiply each of the other terms by 300.

Meredith's share:Raschid's share:total

$$\begin{array}{ccccc} 2 & : & 3 & : & 5 \\ \downarrow \times 300 & & \downarrow \times 300 & & \downarrow \times 300 \\ = 600 & : & 900 & : & 1500 \end{array}$$

Meredith received \$600. Raschid received \$900.

13. **Step 1:** Write a proportion. Let $\boxed{}$ be the number of bones in the head. **Note:** This ratio is approximate.

bones in head:bones in body

$$\begin{aligned} &1 : 7 \\ = &\boxed{} : 206 \end{aligned}$$

Step 2: Find the missing term. Use this reasoning. Because 206 was multiplied by about 29 to get 206, multiply the other term by 29.

$$\begin{array}{ccc} 1 & : & 7 \\ \downarrow \times 29 & & \downarrow \times 29 \\ \doteq 29 & : & 206 \end{array}$$

$206 \div 7 \doteq 29$

There are about 29 bones in an adult's head.

14. **Step 1:** Express the ratio of cans of concentrate to cans of water as a single number.

$\dfrac{\text{water}}{\text{concentrate}}$

$$\frac{3}{1} = 3$$

Step 2: Write a formula that shows how the number of cans of water is related to the number of cans of concentrate.

Let w be the number of cans of water.
Let c be the number of cans of concentrate.

$$w = 3c$$

Step 3: Calculate the number of cans of water required for 2 cans of concentrate.

$$\begin{aligned} w &= 3c \\ &= 3(2) \\ &= 6 \end{aligned}$$

You would add 6 cans of water to 2 cans of concentrate.

15. **Step 1:** Express the ratio of white vinegar to vegetable oil as a single number.

$$\frac{\text{white vinegar}}{\text{vegetable oil}} \qquad \frac{1}{2} = 0.5$$

Step 2: Write a formula that shows how the amount of white vinegar is related to the amount of vegetable oil.

Let w represent the amount (in mL) of white vinegar.
Let v represent the amount (in mL) of vegetable oil.

$$w = 0.5\,v$$

Step 3: Calculate the amount of white vinegar required for 60 mL of vegetable oil.

$$\begin{aligned} w &= 0.5\,v \\ &= 0.5(60) \\ &= 30 \end{aligned}$$

You use 30 mL of white vinegar to 60 mL of vegetable oil.

16. **Step 1:** Express the ratio of uncooked pasta to cooked pasta as a single number.

$$\frac{\text{uncooked}}{\text{cooked}} \qquad \frac{1}{3}$$

Step 2: Write a formula that shows how the amount of uncooked pasta is related to the amount of cooked pasta.

Let u represent the amount (in mL) of uncooked pasta.
Let c represent the amount (in mL) of cooked pasta.

$$u = \frac{1}{3}c$$

Step 3: Calculate the amount of uncooked pasta you would use in order to make 900 mL of cooked pasta.

$$\begin{aligned} u &= \frac{1}{3}c \\ &= \frac{1}{3}(900) \\ &= 300 \end{aligned}$$

You use 300 mL of uncooked pasta.

Now Try This

17. Use logical and proportional reasoning to solve the problem.

Step 1: Write the ratio of unburned toast to total toast.

unburned:total   4:5

Step 2: Calculate the total number of slices of unburned toast required for 90 people if each person receives 2 slices.

$$2 \times 90 = 180$$

Step 3: Write a proportion.

unburned:total $4 : 5$
$= 180 : \boxed{}$

Step 4: Find the missing term. Use this reasoning. Because 4 was multiplied by 45 to get 180, multiply the other term by 45.

$$\begin{array}{ccc} 4 & : & 5 \\ \downarrow \times 45 & & \downarrow \times 45 \\ = 180 & : & \boxed{225} \end{array}$$

The required number of slices is 225.

18. Make an organized list and use integers to show the transactions.

Jane	$(+10)+(-1)+(+3)=12$
Gilbert	$(+10)+(+1)+(+2)+(-3)+(+2)=12$
Tanya	$(+10)+(-1)+(-5)+(-2)+(+1)=3$
Katherine	$(+10)+(+1)+(-2)+(+1)+(-1)=9$
Lloyd	$(+10)+(+3)+(+5)+(-1)+(-3)=14$

Lloyd has the most money. Tanya has the least money.

Section 1: Activity 2

1. **Step 1:** Write the hourly earnings (in \$) for Marlene.

$$\frac{\text{total earnings (\$)}}{\text{total time (h)}} \qquad \frac{44}{8} = 5.5$$

Marlene earned \$5.50 per hour.

Step 2: Write the hourly earnings (in \$) for Jeannie.

$$\frac{\text{total earnings (\$)}}{\text{total time (h)}} \qquad \frac{39}{6} = 6.5$$

Jeannie earned \$6.50 per hour.

Step 3: Compare the rates.

$6.50 > 5.50$

Jeannie earned the better rate of pay.

2. **Step 1:** Write the rate of speed (in km/h) for Arvind.

$$\frac{\text{total distance (km)}}{\text{total time (h)}} \qquad \frac{740}{8} = 92.5$$

Arvind travelled at an average speed of 92.5 km/h.

Step 2: Write the rate of speed (in km/h) for Tim.

$$\frac{\text{total distance (km)}}{\text{total time (h)}} \qquad \frac{650}{7} \doteq 92.9$$

Tim travelled at an average rate of speed of 92.9 km/h.

Step 3: Compare the rates.

$92.9 > 92.5$

Tim travelled at a faster rate of speed.

3. **a.** $$\frac{\text{mass (g)}}{\text{volume}\left(\text{cm}^3\right)} \qquad \frac{40}{40} = 1$$

The density of water is $1\ \text{g/cm}^3$.

b. $$\frac{\text{mass (g)}}{\text{volume}\left(\text{cm}^3\right)} \qquad \frac{350}{50} = 7$$

The density of cast iron is $7\ \text{g/cm}^3$.

c. $$\frac{\text{mass (g)}}{\text{volume}\left(\text{cm}^3\right)} \qquad \frac{40}{60} = 0.\dot{6}$$

The density of oak is $0.\dot{6}\ \text{g/cm}^3$.

d. $7 > 1 > 0.\dot{6}$

Cast iron is denser than water, which is denser than oak. Therefore, the cast iron is the densest material.

e. Objects with a density of less than $1\ \text{g/cm}^3$ can float. For example, a piece of oak can float, but a solid piece of cast iron cannot.

4. **a.**

	Bailey	Johnson
$\dfrac{\text{distance (m)}}{\text{time (s)}}$	$\dfrac{100}{9.84}$	$\dfrac{200}{19.32} \underset{\div 2}{\overset{\div 2}{=}} \dfrac{100}{9.66}$

$9.66 < 9.84$

In 1996, Johnson appeared to be the faster runner.

b. Answers will vary. The article explains that you cannot simply divide Michael Johnson's time by two. In the first 100 m of a 200-m race the runner is accelerating; during the second 100 m, the runner is cruising.

It was reported that Michael Johnson ran the first 100 m in 10.12 s and his second 100 m in 9.20 s.

$10.12 > 9.84$

Donovan Bailey was faster.

5. **a.**

$$\frac{\text{total cost (\$)}}{\text{total mass (g)}} \qquad \frac{4.47}{300} \underset{\div 3}{\overset{\div 3}{=}} \frac{1.49}{100}$$

The salami cost $1.49 per 100 g.

b.

$$\frac{\text{total cost (\$)}}{\text{total mass (g)}} \qquad \frac{3.75}{250} \underset{\div 2.5}{\overset{\div 2.5}{=}} \frac{1.5}{100}$$

The black forest ham cost $1.50 per 100 g.

6. **a.**

$$\frac{\text{marriages}}{\text{population}} \qquad \frac{184\,096}{25\,942\,000} \underset{\div 25\,942}{\overset{\div 25\,942}{\doteq}} \frac{7.1}{1000}$$

In 1985 the marriage rate was about 7.1 per 1000 people.

b.

$$\frac{\text{marriages}}{\text{population}} \qquad \frac{160\,616}{29\,606\,000} \underset{\div 29\,606}{\overset{\div 29\,606}{\doteq}} \frac{5.4}{1000}$$

In 1995 the marriage rate was about 5.4 per 1000 people.

7. **Step 1:** Write a proportion.

$$\frac{\text{words typed}}{\text{time (min)}} \qquad \frac{125}{1} = \frac{2000}{\square}$$

Step 2: Find the missing term.

$$\frac{\text{words typed}}{\text{time (min)}} \qquad \frac{125}{1} \underset{\times 16}{\overset{\times 16}{=}} \frac{2000}{\boxed{16}}$$

Samuel can type the report in 16 min.

8. a. **Step 1:** Write a proportion.

$$\frac{\text{cost (\$)}}{\text{mass (g)}} \qquad \frac{2.25}{100} = \frac{\boxed{}}{400}$$

Step 2: Find the missing term.

$$\frac{\text{cost (\$)}}{\text{mass (g)}} \qquad \frac{2.25}{100} \underset{\times 4}{\overset{\times 4}{=}} \frac{\boxed{9}}{400}$$

It would cost \$9.00.

b. **Step 1:** Write a proportion.

$$\frac{\text{cost (\$)}}{\text{mass (g)}} \qquad \frac{2.25}{100} = \frac{20}{\boxed{}}$$

Step 2: Find the missing term.

$$\frac{\text{cost (\$)}}{\text{mass (g)}} \qquad \frac{2.25}{100} \underset{\times 8.9}{\overset{\times 8.9}{\doteq}} \frac{20}{\boxed{890}}$$

You could buy about 890 g of smoked turkey.

9. **DC10**

$$\begin{aligned} d &= st \\ &= 880(3) \\ &= 2640 \end{aligned}$$

The DC10 will fly 2640 km.

B747

$$\begin{aligned} d &= st \\ &= 920(3) \\ &= 2760 \end{aligned}$$

The B747 will fly 2760 km.

B737

$$\begin{aligned} d &= st \\ &= 840(3) \\ &= 2520 \end{aligned}$$

The B737 will fly 2520 km.

10. **Step 1:** Write a formula. Let d be the distance travelled (in km), s be the speed (in km/min), and t be the time (in min).

$$d = st$$

Step 2: Determine the distance travelled in 3 h.

$$\begin{aligned} d &= st \\ &= 0.005(180) \\ &= 0.9 \end{aligned}$$

1 h = 60 min
3 h = 180 min

The tortoise will travel 0.9 km in 3 h.

11. **Step 1:** Write a formula. Let h be the number of heartbeats. Let t be the time (in min).

$$h = 72t$$

Step 2: Determine the number of heartbeats in 30 s.

$$\begin{aligned} h &= 72t \\ &= 72(0.5) \\ &= 36 \end{aligned}$$

60 s = 1 min
30 s = 0.5 min

The heart will beat 36 times in 30 s.

12. **Step 1:** Write a formula. Let b be the number of blinks and t be the time (in min).

$$b = 25t$$

Step 2: Determine the number of blinks in 1 h.

$$\begin{aligned} b &= 25t \\ &= 25(60) \\ &= 1500 \end{aligned}$$

1 h = 60 min

Your eye blinks about 1500 times in 1 h.

13. If you put all the eggs in one pot, it will take 3 min to boil six eggs.

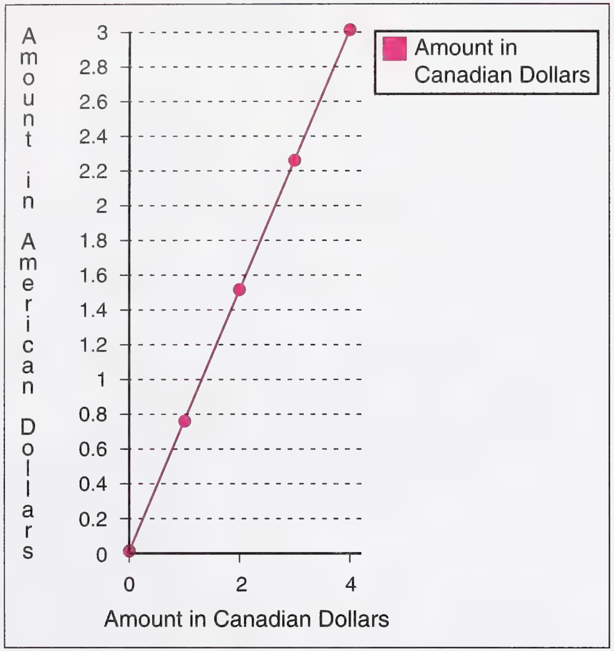
14. Sylvia assumed the oven temperature would be doubled as well as all the ingredients. The cookies should be baked at 350°F.
Note: The baking time would not double.

15. **a.**

	A	B
1	Amount in Canadian Dollars	Amount in American Dollars
2	$0.00	$0.00
3	$1.00	$0.75
4	$2.00	$1.50
5	$3.00	$2.25
6	$4.00	$3.00

Note: Experiment with your spreadsheet or read the computer manual and discover how to express the numbers in dollars and cents.

b.

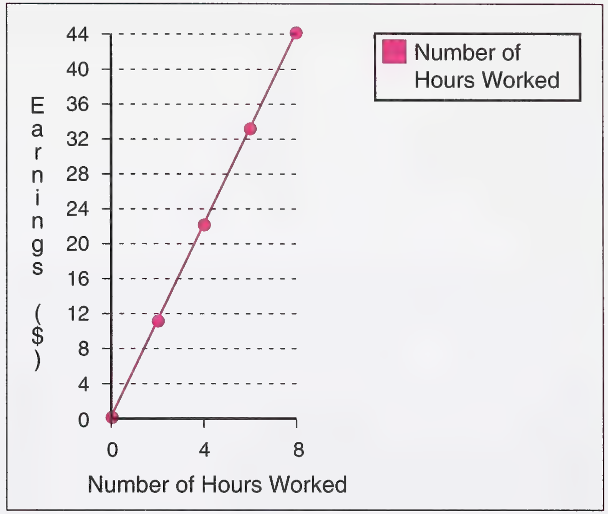


c. Yes, the situation is proportional. The graph is a straight line passing through the origin and leaning upward and to the right.

16. a.

	A	B
1	Number of Hours Worked	Earnings ($)
2	0	0
3	2	11
4	4	22
5	6	33
6	8	44

b.

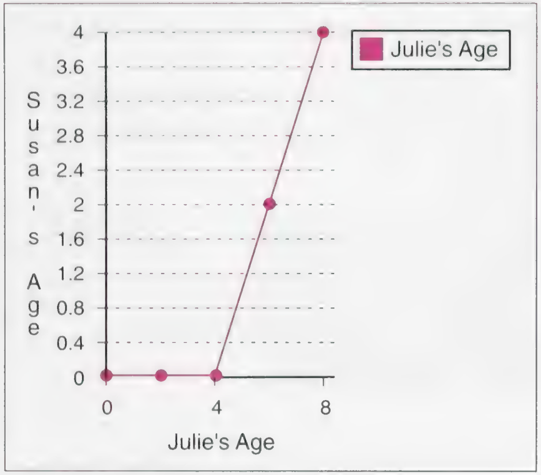


c. Yes, this is a proportional situation. The graph is a straight line passing through the origin and leaning upwards and to the right.

17. a.

	A	B
1	Julie's Age	Susan's Age
2	0	0
3	2	0
4	4	0
5	6	2
6	8	4

b.

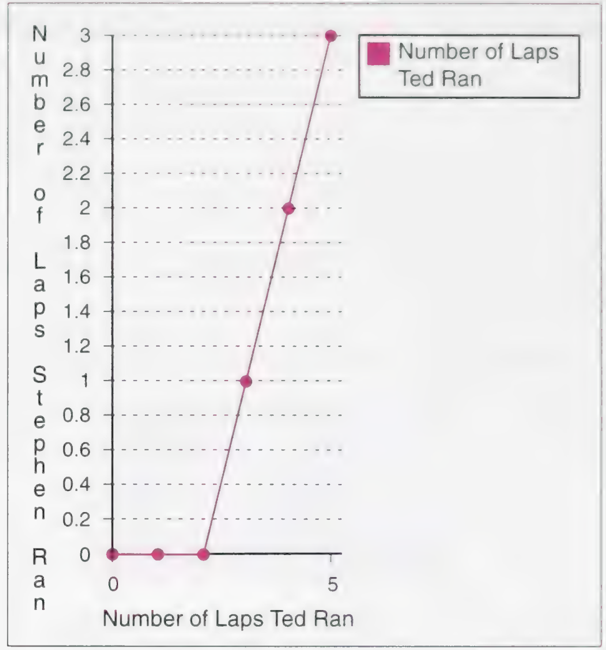


c. No, the situation is **not** proportional. The graph is **not** a straight line passing through the origin and leaning upward and to the right; the graph is a broken-line graph.

18. a.

	A	B
1	Number of Laps Ted Ran	Number of Laps Stephen Ran
2	0	0
3	1	0
4	2	0
5	3	1
6	4	2
7	5	3

b.

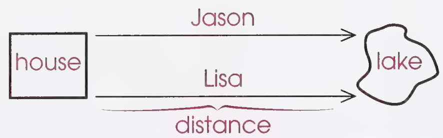


c. No, the situation is **not** proportional. The graph is **not** a straight line passing through the origin and leaning upward and to the right; the graph is a broken-line graph.

Now Try This

19. You can use diagrams and the guess, check, and revise method to help you solve this question.

Step 1: Draw a diagram.



Step 2: Organize the data in a table. (In the table, let d be the distance (in km), s be the speed (in km/h), and t be the time (in h).

	d	s	t
Jason		6	
Lisa		18	

Step 3: Guess the time that it took for each person to travel to the lake. Remember that Lisa travelled for one hour less than Jason.

Guess 1: Lisa travelled for 2 h; Jason travelled for 3 h. Calculate the distances using the formula

$d = st$

	d	s	t
Jason	18	6	3
Lisa	36	18	2

Check to see if the guess is correct. Are the distances equal? No, $18 \neq 36$.

Guess 2: Lisa travelled for 3 h; Jason travelled for 4 h. Calculate the distances using the formula $d = st$.

	d	s	t
Jason	24	6	4
Lisa	54	18	3

Check to see if the guess is correct. Are the distances equal? No, $24 \neq 54$

Guess 3: Lisa travelled for 0.5 h; Jason travelled for 1.5 h. Calculate the distances using the formula $d = st$.

	d	s	t
Jason	9	6	1.5
Lisa	9	18	0.5

Check to see if the guess is correct. Are the distances equal? Yes, $9 = 9$.

The lake was 9 km from the house.

Section 1: Activity 3

1. a. **Step 1:** Express the ratio as a decimal number.

$$\frac{6}{12} = 0.5$$

Step 2: Write an equivalent ratio with a second term of 100.

$$\frac{6}{12} = 0.5 = \frac{50}{100}$$

Step 3: Write the percent.

$$\frac{6}{12} = 0.5 = \frac{50}{100} = 50\%$$

Ron has 50% of the chocolate bar.

b. **Step 1:** Write the ratio as a decimal number.

$$\frac{4}{12} = 0.\dot{3}$$

Step 2: Write an equivalent ratio with a second term of 100.

$$\frac{4}{12} = 0.\dot{3} = \frac{33.\dot{3}}{100}$$

Step 3: Write the percent.

$$\frac{4}{12} = 0.\dot{3} = \frac{33.\dot{3}}{100} = 33.\dot{3}\%$$

Ron has $33.\dot{3}\%$ of the chocolate bar.

c. **Step 1:** Write the ratio as a decimal number.

$$\frac{15}{12} = 1.25$$

Step 2: Write an equivalent ratio with a second term of 100.

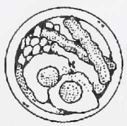
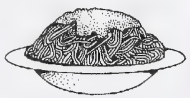
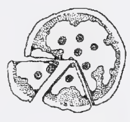
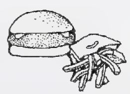
$$\frac{15}{12} = 1.25 = \frac{125}{100}$$

Step 3: Write the percent.

$$\frac{15}{12} = 1.25 = \frac{125}{100} = 125\%$$

Ron has 125% of the chocolate bar.

2. a.

Serving	Amount of Sodium per Serving (mg)	Percentage of Recommended Maximum Daily Amount (%)
 cereal and milk	225	$\frac{225}{2400} \doteq 0.09$ $\doteq \frac{9}{100}$ $\doteq 9\%$
 bacon, eggs, and hashbrowns	1000	$\frac{1000}{2400} \doteq 0.42$ $\doteq \frac{42}{100}$ $\doteq 42\%$
 pasta and sauce	400	$\frac{400}{2400} \doteq 0.17$ $\doteq \frac{17}{100}$ $\doteq 17\%$
 personal pizza	1100	$\frac{1100}{2400} \doteq 0.46$ $\doteq \frac{46}{100}$ $\doteq 46\%$
 burger and fries	1000	$\frac{1000}{2400} \doteq 0.42$ $\doteq \frac{42}{100}$ $\doteq 42\%$

b. From the chart in 2.a., find the percentages that correspond to the foods Vern ate and add the percentages.

$$42\% + 46\% + 42\% = 130\%$$

Vern ate 130% of the recommended maximum daily amount of sodium.

c. From the chart in 2.a., find the percentages that correspond to the foods Deepa ate and add the percentages.

$$9\% + 46\% + 17\% = 72\%$$

Deepa ate 72% of the recommended maximum daily amount of sodium.

Note: For questions 2.b. and 2.c. you could also add the individual amounts of sodium consumed and then calculate the percentage of the recommended maximum daily amount.

d. $130\% > 72\%$

Deepa consumed less sodium that day.

e. Vern consumed more than the daily maximum.

3. a. $400\% = 400 \div 100$

$= 4$

The area of Lake Superior is 4 times the area of Lake Ontario.

b. $156\% = 156 \div 100$

$= 1.56$

The mass of 1 L of milk is 1.56 times the mass of 1 L of gasoline.

c. $87.5\% = 87.5 \div 100$

$= 0.875$

Raymond won 0.875 of the races in which he competed.

d. $16\frac{2}{3}\% = 16\frac{2}{3} \div 100$

$= \frac{50}{3} \div 100$

$= \frac{50}{3} \times \frac{1}{100}$

$= \frac{\overset{1}{\cancel{50}} \times 1}{3 \times \underset{2}{\cancel{100}}}$

$= \frac{1}{6}$

The flying speed of the spine-tail swift is $\frac{1}{6}$ of the flying speed of some jet planes.

4. Method 1: Using a Proportion

Step 1: Write a proportion.

commission:selling price

$45 : 1500$

$= \boxed{} : 100$

Step 2: Find the missing value.

commission:selling price

$45 : 1500$

$\div 15 \quad \div 15$

$= \boxed{3} : 100$

Step 3: Write the percent.

$3 : 100 = 3\%$

Rhonda earned a commission of 3%.

Method 2: Using a Formula

Step 1: Write a formula.

Let c be the commission (in dollars).
Let r be the rate of commission (as a decimal number).
Let s be the selling price (in dollars).

$c = rs$

Step 2: Substitute the given values into the formula.

$$c = rs$$
$$45 = r(1500)$$

Step 3: To solve for r, divide each side of the equation by 1500.

$$\frac{45}{1500} = \frac{r(1500)}{1500}$$
$$0.03 = r$$

Step 4: Express the rate as a percent.

$$\begin{aligned} 0.03 &= \frac{3}{100} \\ &= 3\% \end{aligned}$$

Rhonda earned a commission of 3%.

5. **Method 1: Using a Proportion**

Step 1: Write the rate with a second term of 100.

$$3\% = \frac{3}{100}$$

Step 2: Write a proportion.

$$\frac{\text{commission}}{\text{selling price}} \qquad \frac{3}{100} = \frac{\square}{3500}$$

Step 3: Find the missing value.

$$\frac{\text{commission}}{\text{selling price}} \qquad \frac{3}{100} \overset{\times 35}{\underset{\times 35}{=}} \frac{105}{3500}$$

Mr. Ristoff earned $105 in commission.

Method 2: Using a Formula

Step 1: Write a formula.

Let c be the commission (in dollars).
Let r be the rate (as a decimal number).
Let s be the selling price (in dollars).

Step 2: Write the rate as a decimal number.

$$\begin{aligned} 3\% &= 3 \div 100 \\ &= 0.03 \end{aligned}$$

Step 3: Substitute the given values in the formula.

$$\begin{aligned} c &= rs \\ &= 0.03(3500) \\ &= 105 \end{aligned}$$

Mr. Ristoff earned $105 in commission.

6. **Method 1: Using a Proportion**

Step 1: Write the rate with a second term of 100.

$$1.25\% = \frac{1.25}{100}$$

Step 2: Write a proportion.

$$\frac{\text{commission}}{\text{selling price}} \qquad \frac{1.25}{100} = \frac{\boxed{}}{15\,000}$$

Step 3: Find the missing value.

$$\frac{\text{commission}}{\text{selling price}} \qquad \frac{1.25}{100} \overset{\times 150}{\underset{\times 150}{=}} \frac{\boxed{187.5}}{15\,000}$$

Ms. Carlos earned $187.50 in commission.

Method 2: Using a Formula

Step 1: Write a formula.

Let c be the commission (in dollars).
Let r be the rate (as a decimal number).
Let s be the selling price (in dollars).

Step 2: Express the rate as a decimal number.

$$\begin{aligned} 1.25\% &= 1.25 \div 100 \\ &= 0.0125 \end{aligned}$$

Step 3: Substitute the given values in the formula.

$$\begin{aligned} c &= rs \\ &= 0.0125(15\,000) \\ &= 187.5 \end{aligned}$$

Ms. Carlos earned $187.50 in commission.

7. **Step 1:** Write the percent as a decimal number.

$$\begin{aligned} 12\frac{1}{4}\% &= 12.25\% \\ &= 12.25 \div 100 \\ &= 0.1225 \end{aligned}$$

Step 2: Calculate the interest.

$$\begin{aligned} I &= Prt \\ &= 14\,000(0.1225)(1) \\ &= 1715 \end{aligned}$$

Mr. Marston paid $1715 in interest.

8. **a. Step 1:** Express the time in years.

$$\begin{aligned} 1\text{ mo} &= \frac{1}{12}\text{ a} \\ \therefore\ 3\text{ mo} &= \frac{3\text{a}}{12} \\ &= 0.25\text{ a} \end{aligned}$$

Step 2: Express the %/a as a decimal number.

$$\begin{aligned} 5\% &= 5 \div 100 \\ &= 0.05 \end{aligned}$$

Step 3: Calculate the interest.

$$\begin{aligned} I &= Prt \\ &= 3000(0.05)(0.25) \\ &= 37.5 \end{aligned}$$

The interest is \$37.50.

b. Step 1: Express the percent as %/a.

$$\begin{aligned} 1\%/\text{mo} &= 12\%/\text{a} \\ \therefore\ 1\tfrac{1}{2}\%/\text{mo} &= 18\%/\text{a} \end{aligned}$$

Step 2: Express the %/a as a decimal number.

$$\begin{aligned} 18\% &= 18 \div 100 \\ &= 0.18 \end{aligned}$$

Step 3: Calculate the interest.

$$\begin{aligned} I &= Prt \\ &= 450(0.18)(1) \\ &= 81 \end{aligned}$$

The interest is \$81.

c. Step 1: Express the time in years.

$$\begin{aligned} 1\text{ mo} &= \frac{1}{12}\text{a} \\ \therefore\ 6\text{ mo} &= \frac{6}{12}\text{a} \\ &= 0.5\text{ a} \end{aligned}$$

Step 2: Express the percent as %/a.

$$\begin{aligned} 1\%/\text{mo} &= 12\%/\text{a} \\ \therefore\ 2\%/\text{mo} &= 24\%/\text{a} \end{aligned}$$

Step 3: Express the %/a as a decimal number.

$$\begin{aligned} 24\% &= 24 \div 100 \\ &= 0.24 \end{aligned}$$

Step 4: Calculate the interest.

$$\begin{aligned} I &= Prt \\ &= 1500(0.24)(0.5) \\ &= 180 \end{aligned}$$

The interest is \$180.

9. a. The formula =1500*0.12 was entered in cell B2 to calculate the simple interest. **Note:** $I = Prt$

   b. The formula =1500+B2 was entered in cell C2 to calculate the amount after one year. **Note:** $A = P + I$

   c. The formula =C2*0.12 was entered in cell B3 to calculate the simple interest. **Note:** $I = Prt$

   d. The formula =C2+B3 was entered in cell C3 to calculate the amount after two years. **Note:** $A = P + I$

10. These are the data and formulas you enter.

	A	B	C
1	End of Year	Interest	Amount
2	1	=1500*0.18	=1500+B2
3	2	=C2*0.18	=C2+B3
4	3	=C3*0.18	=C3+B4
5	4	=C4*0.18	=C4+B5
6	5	=C5*0.18	=C5+B6
7	6	=C6*0.18	=C6+B7

Mrs. Kowalski will have \$4049.33 after six years.

11. a. **Step 1:** Calculate the GST.

$$\begin{aligned} 7\% \text{ of } 600 &= 0.07 \times 600 \\ &= 42 \end{aligned}$$

The GST is \$42.

Step 2: Find the total cost.

$$600 + 42 = 642$$

The snowboard cost \$642 in Alberta.

   b. **Step 1:** Calculate the GST.

$$\begin{aligned} 7\% \text{ of } 600 &= 0.07 \times 600 \\ &= 42 \end{aligned}$$

The GST is \$42.

Step 2: Calculate the PST.

$$\begin{aligned} 7\% \text{ of } 600 &= 0.07 \times 600 \\ &= 42 \end{aligned}$$

The PST is \$42.

Step 3: Find the total cost.

$$600 + 42 + 42 = 684$$

The snowboard cost \$684 in Manitoba.

c. **Step 1:** Calculate the GST.

$$\begin{aligned} 7\% \text{ of } 600 &= 0.07 \times 600 \\ &= 42 \end{aligned}$$

The GST is \$42.

Step 2: Calculate the PST.

$$\begin{aligned} 6.5\% \text{ of } 600 &= 0.065 \times 600 \\ &= 39 \end{aligned}$$

The PST is \$39.

Step 3: Find the total cost.

$$600 + 42 + 39 = 681$$

The snowboard cost \$681 in British Columbia.

12. **Step 1:** Write the percent as a decimal.

$$\begin{aligned} 25\% &= 25 \div 100 \\ &= 0.25 \end{aligned}$$

Step 2: Calculate the discount.

$$\begin{aligned} 25\% \text{ of } 80 &= 0.25 \times 80 \\ &= 20 \end{aligned}$$

The discount is \$20.

Step 3: Calculate the sale price.

$$80 - 20 = 60$$

The sale price is \$60.

13. **Step 1:** Express the percent as a decimal.

$$\begin{aligned} 12.5\% &= 12.5 \div 100 \\ &= 0.125 \end{aligned}$$

Step 2: Calculate the discount.

$$\begin{aligned} 12.5\% \text{ of } 240 &= 0.125 \times 240 \\ &= 30 \end{aligned}$$

The discount is \$30.

Step 3: Calculate the sale price.

$$240 - 30 = 210$$

The sale price is \$210.

14. These are the data and formulas you enter.

	A	B	C
1	Week	Discount	Sale Price
2	1	=1500*0.1	=1500-B2
3	2	=C2*0.1	=C2-B3
4	3	=C3*0.1	=C3-B4
5	4	=C4*0.1	=C4-B5

The final sale price was \$984.15.

15. **Method 1: Doubling the GST**

$2 \times 2.45 \doteq 5$

The ladies should give a tip of about \$5.

Method 2: Rounding Up and Down

Rounding Down	Rounding Up
$35 \doteq 30$	$35 \doteq 40$
15% of 30	15% of 40
$= 0.15 \times 30$	$= 0.15 \times 40$
$= 4.50$	$= 6$

The ladies should give a tip that is between \$4.50 and \$6.

Now Try This

16. Use logical reasoning to help you solve the problem.

Step 1: Add the percentages given to Astrid, Jessie, and Bernard.

$30\% + 25\% + 20\% = 75\%$

The three children received 75% of the cookies.

Step 2: Calculate the percentage given to Clayton.

$100\% - 75\% = 25\%$

Clayton received 25% of the cookies.

Step 3: Write a proportion.

$$\frac{25}{100} = \frac{5}{\boxed{}}$$

Step 4: Find the missing term.

$$\frac{25}{100} \xrightarrow{\div 5} \frac{5}{\boxed{20}}$$

Mr. Haidar made 20 cookies.

Section 1: Activity 4

1. **Step 1:** Write a proportion.

model:real

$60 : 1$

$= 4.5 : \boxed{}$

Step 2: Calculate the length of a real praying mantis.

model:real

$$\begin{array}{ccc} 60 & : & 1 \\ \downarrow \times 0.075 & & \downarrow \times 0.075 \\ = 4.5 & : & \boxed{0.075} \end{array}$$

The length of a real praying mantis is 0.075 m.

Step 3: Calculate the actual length in centimetres.

$$1\ \text{m} = 100\ \text{cm}$$
$$0.075\ \text{m} = 7.5\ \text{cm}$$

The length of an actual praying mantis is 7.5 cm.

2. **Step 1:** Write a proportion.

model:actual

$$1 : 16$$
$$= 11.5 : \boxed{}$$

Step 2: Calculate the diameter of the actual tire.

model:actual

$$\begin{array}{ccc} 1 & : & 16 \\ \downarrow \times 11.5 & & \downarrow \times 11.5 \\ = 11.5 & : & \boxed{184} \end{array}$$

The diameter of the tire is 184 cm.

Step 3: Calculate the actual diameter in metres.

$$100\ \text{cm} = 1\ \text{m}$$
$$184\ \text{cm} = 1.84\ \text{m}$$

The actual diameter of the tire is 1.84 m.

3. **Step 1:** Write a proportion.

model:actual

$$1 : 24$$
$$= 60 : \boxed{}$$

Step 2: Calculate the actual length of the house in centimetres.

model:actual

$$\begin{array}{ccc} 1 & : & 24 \\ \downarrow \times 60 & & \downarrow \times 60 \\ = 60 & : & \boxed{1440} \end{array}$$

The actual length is 1440 cm.

Step 3: Calculate the actual length of the house in metres.

$$100\ \text{cm} = 1\ \text{m}$$
$$\therefore\ 1440\ \text{cm} = 14.4\ \text{m}$$

The actual length of the house is 14.4 m.

4. **Step 1:** Measure the length and width of the television in the drawing.

The length of the television in the drawing is 1.7 cm; the width is 1.2 cm.

Step 2: Write a proportion.

$\frac{\text{drawn}}{\text{actual}}$

Length $\frac{1}{50} = \frac{1.7}{\boxed{}}$ **Width** $\frac{1}{50} = \frac{1.2}{\boxed{}}$

Step 3: Calculate the actual length and width of the television in centimetres.

Length

$$\frac{1}{50} \xrightarrow{\times 1.7} = \frac{1.7}{\boxed{85}}$$

Width

$$\frac{1}{50} \xrightarrow{\times 1.2} = \frac{1.2}{\boxed{60}}$$

The actual length of the television is 85 cm; the actual width is 60 cm.

Step 4: Calculate the length and width of the television in metres.

$$100 \text{ cm} = 1 \text{ m}$$
$$\therefore\ 60 \text{ cm} = 0.6 \text{ m}$$
$$85 \text{ cm} = 0.85 \text{ m}$$

The television is 0.85 m × 0.6 m.

5. **Step 1:** Measure the width of the gate in the drawing.

The width of the gate in the drawing is 0.3 cm.

Step 2: Write a proportion.

drawn (cm):actual (m)

$1 : 15$

$= 0.3 : \boxed{}$

Step 3: Calculate the width of the actual gate.

drawn (cm):actual (m)

$1 : 15$

$\times 0.3 \quad \times 0.3$

$= 0.3 : \boxed{4.5}$

The actual width of the gate is 4.5 m.

6. **Step 1:** Measure the height of the drawn Calgary Tower.

The height is 3 cm.

Step 2: Write a proportion.

drawn:actual

$$1 : 6400$$
$$= 3 : \square$$

Step 3: Calculate the height of the actual Calgary Tower in centimetres.

drawn:actual

$$\begin{array}{ccc} 1 & : & 6400 \\ \downarrow \times 3 & & \downarrow \times 3 \\ = 3 & : & \boxed{19\,200} \end{array}$$

The actual height of the Calgary Tower is 19 200 cm.

Step 4: Calculate the actual height in metres.

$$100 \text{ cm} = 1 \text{ cm}$$
$$\therefore\ 19\,200 \text{ cm} = 192 \text{ m}$$

The actual height of the Calgary Tower is 192 m.

Note: The Calgary Tower is actually 190.8 m high. Because the drawing of the Calgary Tower was reduced so much, there was a slight difference in the calculated height and the actual height.

7. **Step 1:** Measure the diameter of B in the drawing.

The diameter of B in the drawing is 2.8 cm.

Step 2: Write a proportion.

$\frac{\text{drawn}}{\text{actual}}$

$$\frac{1}{10} = \frac{2.8}{\square}$$

Step 3: Calculate the actual diameter of B.

$\frac{\text{drawn}}{\text{actual}}$

$$\frac{1}{10} \overset{\times 2.8}{\underset{\times 2.8}{=}} \frac{2.8}{\boxed{28}}$$

The actual diameter of B is 28 cm.

8. **Step 1:** Measure the length of C in the drawing.

The length of C in the drawing is 8.6 cm.

Step 2: Write a proportion.

$\frac{\text{drawn}}{\text{actual}}$

$$\frac{1}{10} = \frac{8.6}{\square}$$

Step 3: Calculate the actual length of C.

drawn/actual

$$\frac{1}{10} \overset{\times 8.6}{=} \frac{8.6}{\boxed{86}}$$

The actual length of C is 86 cm.

9. a. **Step 1:** On the map, measure the straight-line distance (in centimetres) from Charlottetown to Halifax.

   On the map it is 0.7 cm from Charlottetown to Halifax.

   Step 2: Use the bar scale to write a scale statement.

   Scale 1 cm represents 250 km.

   Step 3: Write a proportion.

   drawn (cm):actual (km)

   $$1 : 250$$
   $$= 0.7 : \boxed{}$$

   Step 4: Calculate the actual straight-line distance.

   drawn (cm):actual (km)

   $$1 : 250$$
   $$\downarrow \times 0.7 \quad \downarrow \times 0.7$$
   $$= 0.7 : \boxed{175}$$

   The actual straight-line distance from Charlottetown to Halifax is 175 km.

b. **Step 1:** On the map, measure the straight-line distance (in centimetres) from Quebec City to St. John's.

   On the map it is 6 cm from Quebec City to St. John's.

   Step 2: Use the bar scale to write a scale statement.

   Scale 1 cm represents 250 km.

   Step 3: Write a proportion.

   drawn (cm):actual (km)

   $$1 : 250$$
   $$= 6 : \boxed{}$$

Step 4: Calculate the actual straight-line distance in kilometres.

drawn (cm):actual (km)

$$\begin{aligned} &1 : 250 \\ &\downarrow \times 6 \quad \downarrow \times 6 \\ = \; &6 : \boxed{1500} \end{aligned}$$

The actual straight-line distance from Quebec City to St. John's is 1500 km.

10. a. **Step 1:** Measure the length of the insect in the drawing.

The insect in the drawing is 7.7 cm long.

Step 2: Use the bar scale to write a scale statement.

Scale 1 cm represents 0.5 mm.

Step 3: Write a proportion.

drawn (cm):actual (mm)

$$\begin{aligned} &1 : 0.5 \\ = \; &7.7 : \boxed{} \end{aligned}$$

Step 4: Calculate the actual length of the insect.

drawn (cm):actual (mm)

$$\begin{aligned} &1 : 0.5 \\ &\downarrow \times 7.7 \quad \downarrow \times 7.7 \\ = \; &7.7 : \boxed{3.85} \end{aligned}$$

The actual length of the insect is 3.85 mm.

b. The scale drawing is an enlargement.

11. **Step 1:** Measure the length (in centimetres) of the drawn shopping cart.

The length of the drawn shopping cart is 1.5 cm.

Step 2: Use the bar scale to write a scale statement.

Scale 1 cm represents 80 cm.

Step 3: Write a proportion.

drawn:actual

$$\begin{aligned} &1 : 80 \\ = \; &1.5 : \boxed{} \end{aligned}$$

Step 4: Calculate the actual length (in centimetres) of the cart.

drawn:actual

$$\begin{array}{rcl} 1 & : & 80 \\ \downarrow \times 1.5 & & \downarrow \times 1.5 \\ = 1.5 & : & \boxed{120} \end{array}$$

The actual length of the cart is 120 cm.

Step 5: Calculate the length (in metres) of the cart.

$$100 \text{ cm} = 1 \text{ m}$$
$$\therefore\ 120 \text{ cm} = 1.2 \text{ m}$$

The actual length of the cart is 1.2 m.

12. **Step 1:** Calculate the length of the enlargement.

$$\begin{aligned} 130\% \text{ of } 6 &= 1.3 \times 6 \\ &= 7.8 \end{aligned}$$

The length will be 7.8 cm.

Step 2: Calculate the width of the enlargement.

$$\begin{aligned} 130\% \text{ of } 4 &= 1.3 \times 4 \\ &= 5.2 \end{aligned}$$

The width will be 5.2 cm.

The enlargement will be 7.8 cm × 5.2 cm.

13. **a.** **Step 1:** Write the ratio in lowest terms.

$\frac{\text{copy}}{\text{actual}}$ $\qquad \frac{4}{8} = \frac{1}{2}$

Step 2: Write the ratio as a percent.

$$\frac{4}{8} = \frac{1}{2} = \frac{50}{100} = 50\%$$

Duncan should select a setting of 50% on the photocopier.

b. **Step 1:** Write the ratio in lowest terms.

$\frac{\text{copy}}{\text{actual}}$ $\qquad \frac{10}{8} = \frac{5}{4}$

Step 2: Write the ratio as a percent.

$$\frac{10}{8} = \frac{5}{4} = \frac{125}{100} = 125\%$$

Duncan should select a setting of 125% on the photocopier.

14.

15. a. Answers will vary. One student obtained the following results.

The original ΔABC had the following dimensions:

$AB = 25$ mm
$BC = 42$ mm
$AC = 50$ mm

- To make $\Delta A'B'C'$, the knot was tied in the centre of the elastic band. $\Delta A'B'C'$ had the following dimensions:

Enlargement	Reduction
$A'B' = 50$ mm	$A'B' = 12$ mm
$B'C' = 85$ mm	$B'C' = 21$ mm
$A'C' = 100$ mm	$A'C' = 25$ mm

The scale factor of the enlargement is about 2. The scale factor of the reduction is about $\frac{1}{2}$ or about 0.5.

$$25 : 42 : 50 \xrightarrow{\times 2,\ \times 2,\ \times 2} \doteq 50 : 85 : 100$$

$$25 : 42 : 50 \xrightarrow{\times 0.5,\ \times 0.5,\ \times 0.5} \doteq 12 : 21 : 25$$

- To make $\Delta A''B''C''$, the knot was tied at about $\frac{2}{3}$ of the length of the elastic band. $\Delta A''B''C''$ had the following dimensions:

Enlargement	Reduction
$A''B'' = 35$ mm	$A''B'' = 16$ mm
$B''C'' = 65$ mm	$B''C'' = 27$ mm
$A''C'' = 72$ mm	$A''C'' = 32$ mm

The scale factor of the enlargement is about $\frac{3}{2}$ or about 1.5. The scale factor of the reduction is about $\frac{2}{3}$ or about 0.67.

$$\begin{array}{ccccc} 25 & : & 42 & : & 50 \\ \downarrow \times 1.5 & & \downarrow \times 1.5 & & \downarrow \times 1.5 \\ \doteq 35 & : & 65 & : & 72 \end{array}$$

$$\begin{array}{ccccc} 25 & : & 42 & : & 50 \\ \downarrow \times 0.67 & & \downarrow \times 0.67 & & \downarrow \times 0.67 \\ \doteq 16 & : & 27 & : & 32 \end{array}$$

- To make $\Delta A'''B'''C'''$, the knot was tied at about $\frac{6}{7}$ of the length of the elastic band. $\Delta A'''B'''C'''$ had the following dimensions:

Enlargement	Reduction
$A'''B''' = 28$ mm	$A'''B''' = 21$ mm
$B'''C''' = 52$ mm	$B'''C''' = 35$ mm
$A'''C''' = 60$ mm	$A'''C''' = 42$ mm

The scale factor of the enlargement is about $\frac{7}{6}$ or about 1.2. The scale factor of the reduction is about $\frac{6}{7}$ or about 0.86.

$$\begin{array}{ccccc} 25 & : & 42 & : & 50 \\ \downarrow \times 1.2 & & \downarrow \times 1.2 & & \downarrow \times 1.2 \\ \doteq 28 & : & 52 & : & 60 \end{array}$$

$$\begin{array}{ccccc} 25 & : & 42 & : & 50 \\ \downarrow \times 0.86 & & \downarrow \times 0.86 & & \downarrow \times 0.86 \\ \doteq 21 & : & 35 & : & 42 \end{array}$$

b. The position of the knot determines the scale factor of the drawn triangle.

Note: Questions 16, 17, and 18 show the line segments connecting the dilatation centre and corresponding vertices.

16. a. Step 1: Find the coordinates of point R'.

Point R has coordinates $(4,5)$. ← Multiply by 2.

Point R' will have coordinates $(8,10)$.

Step 2: Find the coordinates of point S'.

Point S has coordinates $(6,1)$. ← Multiply by 2.

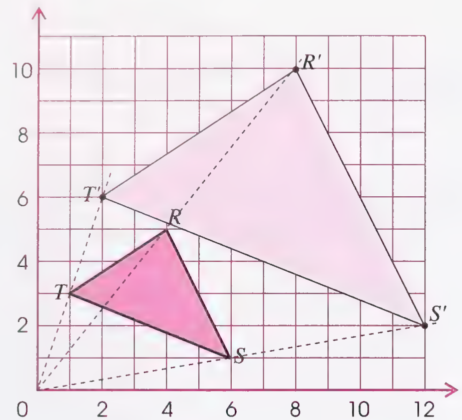
Point S' will have coordinates $(12,2)$.

Step 3: Find the coordinates of point T'.

Point T has coordinates $(1,3)$. ← Multiply by 2.

Point T' will have coordinates $(2,6)$.

Step 4: Plot R', S', and T', and connect the points with line segments.



b. Step 1: Find the coordinates of point L'.

Point L has coordinates $(5,1)$. ← Multiply by 2.

Point L' will have coordinates $(10,2)$.

Step 2: Find the coordinates of point M'.

Point M has coordinates $(2,1)$. ← Multiply by 2.

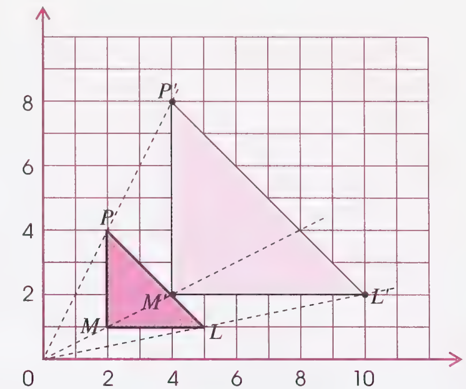
Point M' will have coordinates $(4,2)$.

Step 3: Find the coordinates of point P'.

Point P has coordinates $(2,4)$. ← Multiply by 2.

Point P' will have coordinates $(4,8)$.

Step 4: Plot L', M', and P', and connect the points with line segments.



17. a. Step 1: Find the coordinates of point R'.

Point R has coordinates $(4,5)$. ← Multiply by 0.5.

Point R' will have coordinates $(2,2.5)$.

Step 2: Find the coordinates of point S'.

Point S has coordinates $(6,1)$. ← Multiply by 0.5.

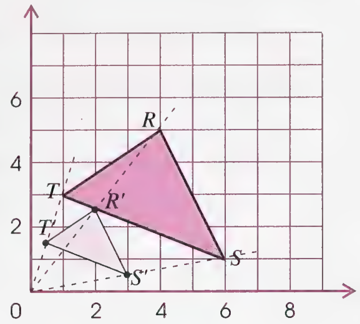
Point S' will have coordinates $(3,0.5)$.

Step 3: Find the coordinates of point T'.

Point T has coordinates $(1,3)$. ← Multiply by 0.5.

Point T' will have coordinates $(0.5,1.5)$.

Step 4: Plot R', S', and T', and connect the points with line segments.



b. Step 1: Find the coordinates of point L'.

Point L has coordinates $(5,1)$. ← Multiply by 0.5.

Point L' will have coordinates $(2.5, 0.5)$.

Step 2: Find the coordinates of point M'.

Point M has coordinates $(2,1)$. ← Multiply by 0.5.

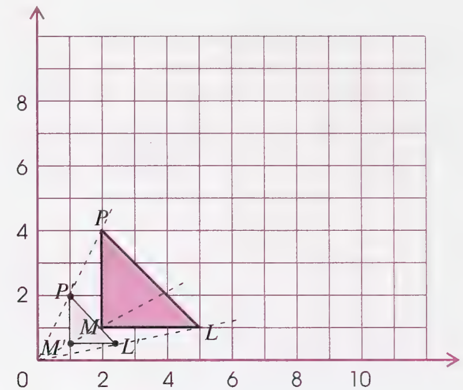
Point M' will have coordinates $(1, 0.5)$.

Step 3: Find the coordinates of point P'.

Point P has coordinates $(2,4)$. ← Multiply by 0.5.

Point P' will have coordinates $(1,2)$.

Step 4: Plot L', M', and P', and connect the points with line segments.



18. a. Step 1: Find the coordinates of point R'.

Point R has coordinates $(4,5)$. ← Multiply by 3.

Point R' will have coordinates $(12,15)$.

Step 2: Find the coordinates of point S'.

Point S has coordinates $(6,1)$. ← Multiply by 3.

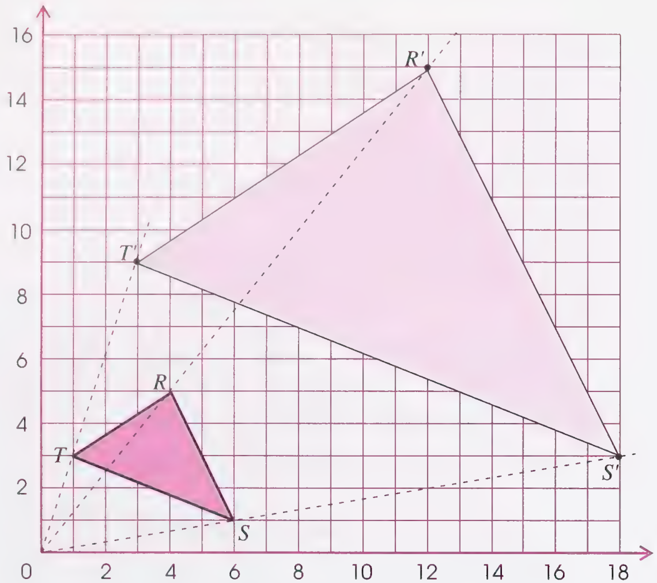
Point S' will have coordinates $(18,3)$.

Step 3: Find the coordinates of point T'.

Point T has coordinates $(1,3)$. ← Multiply by 3.

Point T' will have coordinates $(3,9)$.

Step 4: Plot R', S', and T', and connect the points with line segments.



b. Step 1: Find the coordinates of point L'.

Point L has coordinates $(5, 1)$. ← Multiply by 3.

Point L' will have coordinates $(15, 3)$.

Step 2: Find the coordinates of point M'.

Point M has coordinates $(2, 1)$. ← Multiply by 3.

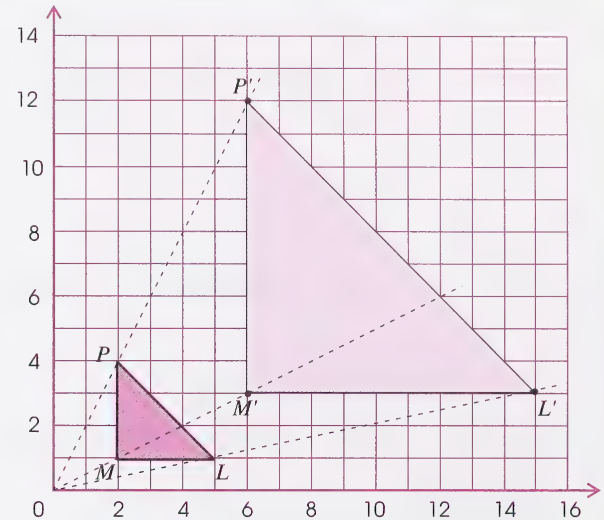
Point M' will have coordinates $(6, 3)$.

Step 3: Find the coordinates of point P'.

Point P has coordinates $(2, 4)$. ← Multiply by 3.

Point P' will have coordinates $(6, 12)$.

Step 4: Plot L', M', and P', and connect the points with line segments.



19. See the preceding diagrams in questions 16, 17, and 18. In each case, when you connect the dilatation centre $(0, 0)$ and the corresponding vertices, the dilatation centre and the corresponding vertices form a straight line.

20. **a.** The IMAX image is over ten times larger than a standard 35-mm frame.

$$\frac{3376}{319} \doteq 10.6$$

The IMAX image is over three times larger than a standard 70-mm frame.

$$\frac{3376}{1072} \doteq 3.15$$

b. A standard 8-mm film would be smaller than 16-mm film. Because the IMAX film is so much larger, the quality of the 8-mm film image would be very poor in comparison.

Section 1: Follow-up Activities

Extra Help

1. 3 0 0 × 7 5 %

225.

The iceberg has a depth of 225 m below water.

2. 1 2 8 × 2 5 0 %

320.

Helmut's expenses were $320.

3. 5 0 0 × 8 7 • 5 %

437.5

There are 437.5 mL of water in the milk.

4. 4 0 0 × 2 • 5 %

10.

In the shipment, 10 telephones were black.

5. **a.** **Method 1: Using Two Steps on the Calculator**

Step 1: Calculate the discount.

9 5 0 × 2 5 %

237.5

The discount is $237.50.

Step 2: Calculate the sale price.

9 5 0 − 2 3 7 • 5 =

712.5

The sale price is $712.50.

Method 2: Using One Step on the Calculator

Taking 25% off means that you pay 75% of the regular price.

100% – 25% = 75%

9 5 0 × 7 5 %

712.5

The sale price is $712.50.

b. **Method 1: Using Two Steps on the Calculator**

Step 1: Calculate the discount.

2 5 0 × 2 0 %

50.

The discount is $50.

Step 2: Calculate the sale price.

2 5 0 – 5 0 =

200.

The sale price is $200.

Method 2: Using One Step on the Calculator

Taking 20% off means that you pay 80% of the regular price.

100% – 20% = 80%

2 5 0 × 8 0 %

200.

The sale price is $200.

6. a. **Method 1: Using Two Steps on the Calculator**

Step 1: Calculate the GST.

9 0 × 7 %

6.3

The GST is $6.30.

Step 2: Calculate the total.

9 0 + 6 . 3 0 =

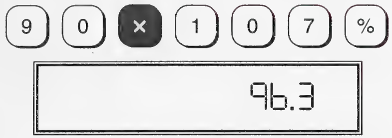
96.3

The total is $96.30.

Method 2: Using One Step on the Calculator

Adding 7% onto the price means that you pay 107% of the price.

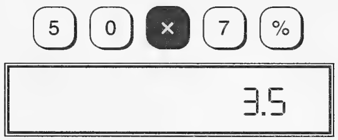
$100\% + 7\% = 107\%$



The total is \$96.30.

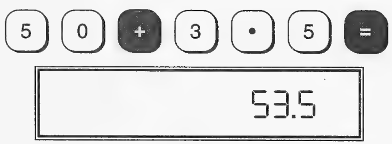
b. **Method 1: Using Two Steps on the Calculator**

Step 1: Calculate the GST.



The GST is \$3.50.

Step 2: Calculate the total price.

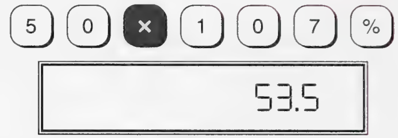


The total is \$53.50.

Method 2: Using One Step on the Calculator

Adding 7% onto the price means that you pay 107% of the price.

$100\% + 7\% = 107\%$



The total is \$53.50.

Enrichment

1. **a.** You can find 10% of 80 and multiply by 8.

$10\% \text{ of } 80 = 8$

$80\% \text{ of } 80 = 64$

b. You can find 10% of 40 and multiply by 3.

$10\% \text{ of } 40 = 4$

$30\% \text{ of } 40 = 12$

2. **a.** You know $33\frac{1}{3}\% = \frac{1}{3}$. You can divide 60 by 3.

$33\frac{1}{3}\% \text{ of } 60 = 20$

b. You can find $33\frac{1}{3}\%$ or $\frac{1}{3}$ of 90 and multiply by 2.

$33\frac{1}{3}\%$ of $90 = 30$

$66\frac{2}{3}\%$ of $90 = 60$

c. You know $25\% = \frac{1}{4}$. You can divide 40 by 4.

25% of $40 = 10$

d. You can find 25% of 80 and divide by 10.

25% of $80 = 20$

2.5% of $80 = 2$

3. **a.** 18% of 50 = 9 **b.** 50% of 18 = 9
c. 88% of 25 = 22 **d.** 25% of 88 = 22

4. The pattern is $a\%$ of b = $b\%$ of a.

5. **a.** 26% of 50
$= 50\%$ of 26

$\therefore$ 26% of $50 = 13$

b. 84% of 25
$= 25\%$ of 84

$\therefore$ 84% of $25 = 21$

c. 55% of 20
$= 20\%$ of 55

$\therefore$ 55% of $20 = 11$

d. 28% of 50
$= 50\%$ of 28

$\therefore$ 28% of $50 = 14$

6. **a.** 45% of 60
$= 0.45 \times 60$
$= 27$

b. 20% of $60 + 25\%$ of 60
$= 0.2 \times 60 + 0.25 \times 60$
$= 12 + 15$
$= 27$

c. 19% of 25
$= 0.19 \times 25$
$= 4.75$

d. 20% of $25 - 1\%$ of 25
$= 0.2 \times 25 - 0.01 \times 25$
$= 5 - 0.25$
$= 4.75$

7. You can break up a percent into parts to make the mental computation easier. Two examples follow:

$45\% = 20\% + 25\%$

$19\% = 20\% - 1\%$

8. **a.** 35% of $80 =$ 25% of $80 + 10\%$ of 80
$\therefore$ 35% of $80 =$ 28

b. 31% of $60 =$ 30% of $60 + 1\%$ of 60
$\therefore$ 31% of $60 =$ 18.6

c. 79% of $50 =$ 80% of $50 - 1\%$ of 50
$\therefore$ 79% of $50 =$ 39.5

d. 19% of $40 =$ 20% of $40 - 1\%$ of 40
$\therefore$ 19% of $40 =$ 7.6

Section 2: Activity 1

1. a. 10, 100, 1000, 10 000, 100 000, 1 000 000, 10 000 000, 100 000 000,…

   b. The number preceding 10 is 1.

2. a. $10^1, 10^2, 10^3, 10^4, 10^5, 10^6, 10^7, 10^8, \ldots$

   b. The power preceding the 10^1 is 10^0.

3. a. The number of zeros that follow 1 in the standard form of each number is equal to the exponent in the corresponding power of ten.

   b. $10^0 = 1$

4. $10\ 000\ 000\ 000 = 10^{10}$

5. $10^{11} = 100\ 000\ 000\ 000$

6. $1\ 000\ 000\ 000\ 000\ 000 = 10^{15}$

7. $10^{25} = 10\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000$

8. a. $2.0 \times 10^7\ °C = 20\ 000\ 000°C$

   b. 9.46×10^{12} km $= 9\ 460\ 000\ 000\ 000$ km

   c. 4.65×10^{31} kg

   $= 46\ 500\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000$ kg

   d. $1.2 \times 10^{11} = 120\ 000\ 000\ 000$

9. a. $40\ 300\ 000\ 000\ 000$ km $= 4.03 \times 10^{13}$ km

   b. $11\ 000$ kg $= 1.1 \times 10^4$ kg

   c. $\$155\ 000\ 000\ 000 = \$1.55 \times 10^{11}$

   d. $70\ 000\ 000\ 000\ 000$ s $= 7.0 \times 10^{13}$ s

10. $\frac{1}{1000}, \frac{1}{100}, \frac{1}{10}$, 1, 10, 100, 1000,…

    or

    0.001, 0.01, 0.1, 1, 10, 100, 1000,…

11. $10^{-3}, 10^{-2}, 10^{-1}, 10^0, 10^1, 10^2, 10^3, \ldots$

12. The powers of ten which correspond to each decimal number have negative exponents. The absolute value of the exponent is equal to the number of decimal places in the standard form of each number.

13. a. 10^{-3} cm $= 0.001$ cm

    b. 10^{-5} cm $= 0.000\ 01$ cm

    10^{-16} g $= 0.000\ 000\ 000\ 000\ 000\ 1$ g

14. a. $0.000\,000\,01 \text{ cm} = 10^{-8} \text{ cm}$

b. $0.000\,000\,000\,001 \text{ cm} = 10^{-12} \text{ cm}$

c. $0.000\,000\,000\,000\,000\,1 \text{ cm} = 10^{-16} \text{ cm}$

15. a. $5.8\times10^{-5} = 5.8\times0.000\,01$

$= 0.000\,058$

$$\begin{array}{r} 1 \text{ place} \\ +\ 5 \text{ places} \\ \hline 6 \text{ places} \end{array}$$

b. $1.28\times10^{-8} = 1.28\times0.000\,000\,01$

$= 0.000\,000\,012\,8$

$$\begin{array}{r} 2 \text{ places} \\ +\ 8 \text{ places} \\ \hline 10 \text{ places} \end{array}$$

c. $7.5\times10^{-3} = 7.5\times0.001$

$= 0.0075$

$$\begin{array}{r} 1 \text{ place} \\ +\ 3 \text{ places} \\ \hline 4 \text{ places} \end{array}$$

16. a. **Step 1:** Write the number 0.000 018. Use a caret to locate the decimal point for a number between 1 and 10.

$0.000\,01{}_{\wedge}8$

The number between 1 and 10 is 1.8.

Step 2: Find the exponent of the power of ten. Remember it will be a negative number.

Count the number of places between the decimal point of 0.000 018 and the caret.

$\underbrace{0.000\,01}_{5 \text{ places}}{}_{\wedge}8$

The exponent of the power of ten is -5.

Step 3: Write the number in scientific notation.

$$0.000\,018 = 1.8\times10^{-5}$$

About 1.8×10^{-5} of the dry air at sea level is neon.

b. **Step 1:** Write the number 0.0003. Use a caret to locate the decimal point for a number between 1 and 10.

$0.0003_{\wedge}$

The number between 1 and 10 is 3.

Step 2: Find the exponent of the power of ten. Remember, it will be a negative number.

Count the number of places between the decimal point of 0.0003 and the caret.

$\underbrace{0.0003}_{4 \text{ places}}{}_{\wedge}$

The exponent of the power of ten is -4.

Step 3: Write the number in scientific notation.

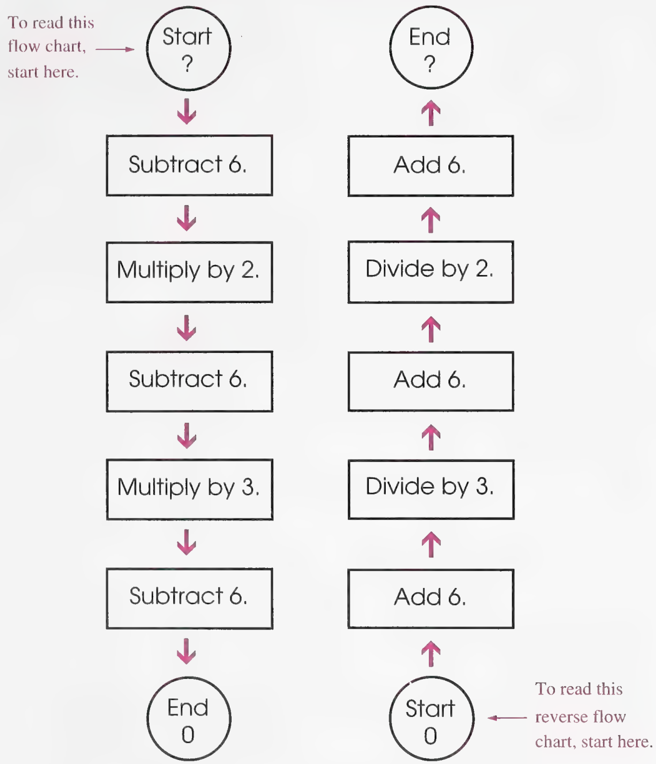
$$0.0003 = 3\times10^{-4}$$

About 3×10^{-4} of all water on Earth evaporates each year.

Now Try This

17. The order of operations in the problem may be shown in a flow chart. To solve the problem, use a reverse flow chart.

Flow Chart **Reverse Flow Chart**



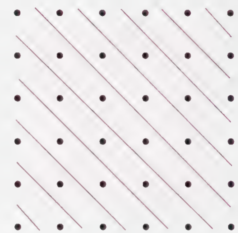
Ten watermelons were put out on Monday morning.

Section 2: Activity 2

1. a.

$5^2 = 1+2+3+4+5$
$+4+3+2+1$

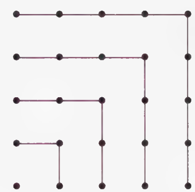
b.



$6^2 = 1+2+3+4+5+6$
$+5+4+3+2+1$

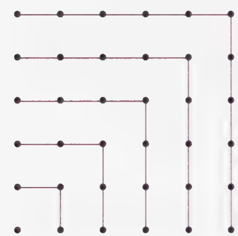
c. $10^2 = 1+2+3+4+5+6+7+8+9+10+9+8+7+6+5$
$+4+3+2+1$

2. a.



$5^2 = 1+3+5+7+9$

b.



$6^2 = 1+3+5+7+9+11$

c. $10^2 = 1+3+5+7+9+11+13+15+17+19$

3. a.

Distance from Screen (units)	Area of Picture (square units)
1	1
2	4
3	9
4	16

b. $a = d^2$

c. $a = d^2$

$= 5^2$

$= 25$

If the projector is 5 units from the screen, the area of the picture on the screen is 25 square units.

4. $d = 5t^2$

$= 5(4)^2$

$= 5(16)$

$= 80$

In 4 s, the object will fall 80 m.

5. a. $d = \frac{s^2}{210}$

$= \frac{50^2}{210}$

$= \frac{2500}{210}$

$\doteq 11.9$

The braking distance is is about 11.9 m.

b. $d = \frac{s^2}{210}$

$= \frac{100^2}{210}$

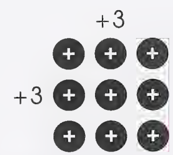
$= \frac{10\,000}{210}$

$\doteq 47.6$

The braking distance is about 47.6 m.

6. View the video to check your answers.

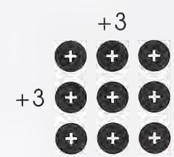
7. a. **Step 1:** To model the principal square root of nine, arrange nine positive counters in a square array.



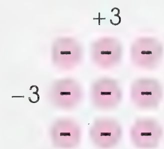
$(+3)\times(+3) = +9$

$\therefore\ \sqrt{9} = +3$

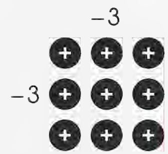
Step 2: To model the negative square root of nine, first arrange nine positive counters in a square array.



To show $(-3)\times(+3)$ exchange the counters for their opposites.



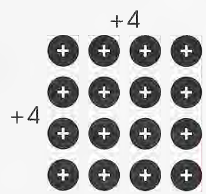
To show $(-3)\times(-3)$ exchange the counters for their opposites.



$(-3)\times(-3)=+9$

$\therefore\ -\sqrt{9}=-3$

b. Step 1: To model the positive square root of 16, arrange 16 positive counters in a square array.



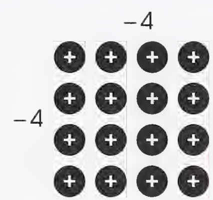
$(+4)\times(+4)=+16$

$\therefore\ \sqrt{16}=+4$

Step 2: To model the negative square root first arrange the 16 positive counters in a square array.

To show $(-4)\times(+4)$ exchange the counters for their opposites.

To show $(-4)\times(-4)$ exchange the counters for their opposites.



$(-4)\times(-4)=+16$

$\therefore\ -\sqrt{16}=-4$

8. $A = s^2$

$64 = s^2$

$s = 8$

$8 \times 8 = 64$

The length of a side of the pool is 8 m.

9. $a = d^2$

$36 = d^2$

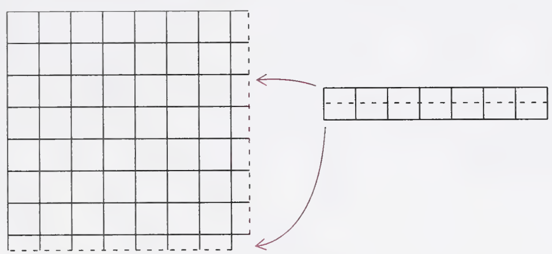
$d = 6$

$6 \times 6 = 36$

The projector is 6 m from the screen.

10. The square root of 42 is about 6.5.

11. a. To show that $\sqrt{56}$ is not a counting number, cut out 56 squares from grid paper. The largest square that can be formed is 7×7. There are 8 squares left over. Cut the 8 leftover squares in half and place them as shown in the following diagram.



b. The square root of 56 is about 7.5.

12. Step 1: Write the formula and substitute the given area.

$$A = s^2$$
$$3000 = s^2$$

Step 2: Solve for s. Since you require a measurement, give only the positive square root.

$$s = \sqrt{3000}$$
$$\doteq 54.8$$

The length of one side of the square field is about 54.8 m.

13. Step 1: Write the formula and substitute the given distance.

$$d = 5t^2$$
$$50 = 5t^2$$

Step 2: Solve for t^2.

$$\frac{50}{5} = \frac{5t^2}{5}$$
$$10 = t^2$$

Step 3: Solve for t. Since you require a measurement, give only the positive square root.

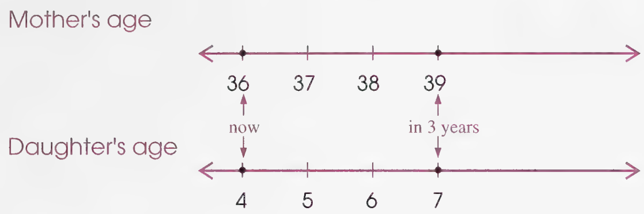
$$t = \sqrt{10}$$
$$\doteq 3.2$$

It took the penny about 3.2 s to hit the water.

Now Try This

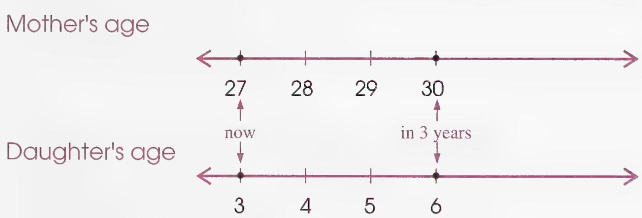
14. You can use number lines and the guess, check, and revise method to answer the problem.

Guess 1: Mrs. Mahr is 36 years old and her daughter is 4. So, in three years Mrs. Mahr will be 39 and her daughter will be 7.



This guess is incorrect because $5 \times 7 \neq 39$.

Guess 2: Mrs. Mahr is 27 years old and her daughter is 3. So, in three years Mrs. Mahr will be 30 and her daughter will be 6.



This guess is correct because $5 \times 6 = 30$.

Mrs. Mahr is now 27 years old and her daughter is now 3.

15. You can use a table and the guess, check, and revise method to solve the problem.

Make a guess and test to see if the guess is correct.

Guess 1

Type of Coin	Number of Coins	Value in Cents
Dimes	10	100
Nickels	30	150
Total		250

The total of 250 cents, or $2.50, is too low. There must be more nickels and dimes.

Guess 2

Type of Coin	Number of Coins	Value in Cents
Dimes	20	200
Nickels	40	200
Total		400

The total of 400 cents, or $4.00, is still too low. There must be more nickels and dimes.

Guess 3

Type of Coin	Number of Coins	Value in Cents
Dimes	50	500
Nickels	70	350
Total		850

The total of 850 cents, or $8.50, is right. Jonathon has 50 dimes and 70 nickels in his piggy bank.

Section 2: Activity 3

1. **a.** Yes, Square B and the four pieces of Square A cover Square C.

   b. The sum of the squares of the legs equals the square of the hypotenuse; that is, $a^2 + b^2 = c^2$.

2. **a.** Yes, Square B and the four pieces of Square A cover Square C.

   b. The sum of the squares of the legs equals the square of the hypotenuse; that is, $a^2 + b^2 = c^2$.

3. **a.** **Step 1:** Write the formula for the Pythagorean relation and substitute the given values. Then simplify the equation.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 35^2 + 12^2 &= c^2 \\ 1225 + 144 &= c^2 \\ 1369 &= c^2 \end{aligned}$$

Step 2: Solve for c. Since a measurement is required, use only a positive square root.

$$\begin{aligned} \sqrt{1369} &= c \\ 37 &= c \end{aligned}$$

Side c is 37 cm long.

b. **Step 1:** Write the formula for the Pythagorean relation and substitute the given values. Then simplify the equation.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 55^2 + b^2 &= 73^2 \\ 3025 + b^2 &= 5329 \end{aligned}$$

Step 2: Solve for b^2.

$$\begin{aligned} \cancel{3025} + b^2 - \cancel{3025} &= 5329 - 3025 \\ b^2 &= 2304 \end{aligned}$$

Step 3: Solve for b. Since a measurement is required, use only a positive square root.

$$b = \sqrt{2304}$$
$$b = 48$$

Side b is 48 cm long.

c. Step 1: Write the formula for the Pythagorean relation and substitute the given values. Then simplify the equation.

$$a^2 + b^2 = c^2$$
$$30^2 + 16^2 = c^2$$
$$900 + 256 = c^2$$
$$1156 = c^2$$

Step 2: Solve for c. Since a measurement is required, use only a positive square root.

$$\sqrt{1156} = c$$
$$34 = c$$

Side c is 34 mm long.

d. Step 1: Write the formula for the Pythagorean relation and substitute the given values. Then simplify the equation.

$$a^2 + b^2 = c^2$$
$$6.3^2 + b^2 = 6.5^2$$
$$39.69 + b^2 = 42.25$$

Step 2: Solve for b^2.

$$\cancel{39.69} + b^2 - \cancel{39.69} = 42.25 - 39.69$$
$$b^2 = 2.56$$

Step 3: Solve for b. Since a measurement is required, use only a positive square root.

$$b = \sqrt{2.56}$$
$$b = 1.6$$

Side b is 1.6 m long.

e. Step 1: Write the formula for the Pythagorean relation and substitute the given values. Then simplify the equation.

$$a^2 + b^2 = c^2$$
$$5.2^2 + 2.0^2 = c^2$$
$$27.04 + 4 = c^2$$
$$31.04 = c^2$$

Step 2: Solve for c. Since a measurement is required, use only a positive square root.

$$\sqrt{31.04} = c$$
$$5.6 \doteq c$$

Side c is about 5.6 km long.

f. Step 1: Write the formula for the Pythagorean relation and substitute the given values. Then simplify the equation.

$$a^2 + b^2 = c^2$$
$$a^2 + 2.0^2 = 2.9^2$$
$$a^2 + 4 = 8.41$$

Step 2: Solve for a^2.

$$a^2 + \cancel{4} - \cancel{4} = 8.41 - 4$$
$$a^2 = 4.41$$

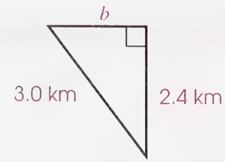
Step 3: Solve for a. Since a measurement is required, use only a positive square root.

$$a = \sqrt{4.41}$$
$$a = 2.1$$

Side a is 2.1 cm long.

4.

$$a^2 + b^2 = c^2$$
$$2.4^2 + b^2 = 3.0^2$$
$$5.76 + b^2 = 9$$
$$\cancel{5.76} + b^2 - \cancel{5.76} = 9 - 5.76$$
$$b^2 = 3.24$$
$$b = 1.8$$

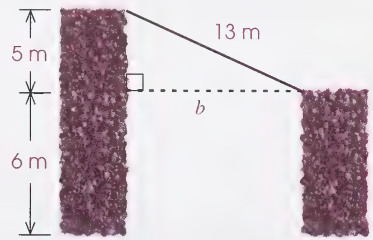


Since a measurement is required, use only a positive square root.

It is 1.8 km from Geeta's cabin to Jack's cottage.

5.

$$a^2 + b^2 = c^2$$
$$5^2 + b^2 = 13^2$$
$$25 + b^2 = 169$$
$$\cancel{25} + b^2 - \cancel{25} = 169 - 25$$
$$b^2 = 144$$
$$b = 12$$

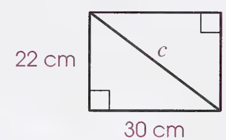


The distance between the walls is 12 m.

Since a measurement is required, use only a positive square root.

6.

$$a^2 + b^2 = c^2$$
$$22^2 + 30^2 = c^2$$
$$484 + 900 = c^2$$
$$1384 = c^2$$
$$c \doteq 37.2$$

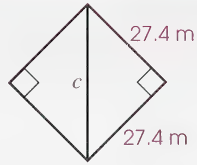


Since a measurement is required, use only a positive square root.

The diagonal measure of the screen is about 37.2 cm.

7.

$$a^2 + b^2 = c^2$$
$$27.4^2 + 27.4^2 = c^2$$
$$750.76 + 750.76 = c^2$$
$$1501.52 = c^2$$
$$38.7 \doteq c$$



The player must throw the ball about 38.7 m.

Since a measurement is required, use only a positive square root.

8.

$$a^2 + b^2 = c^2$$
$$25^2 + 120^2 = c^2$$
$$625 + 14\,400 = c^2$$
$$15\,025 = c^2$$
$$122.6 \doteq c$$

Since a measurement is required, use only a positive square root.

The length of the longest ski pole that could be packed to lie flat is about 122.6 cm.

Did You Know?

9. **a.** $5^2 = 25 \quad 7^2 = 49 \quad 10^2 = 100$

$100 \neq 25 + 49$

No, 5, 7, and 10 is **not** a Pythagorean triple.

b. $7^2 = 49 \quad 24^2 = 576 \quad 25^2 = 625$

$625 = 49 + 576$

Yes, 7, 24, and 25 is a Pythagorean triple.

c. $9^2 = 81 \quad 13^2 = 169 \quad 14^2 = 196$

$196 \neq 81 + 169$

No, 9, 13, and 14 is **not** a Pythagorean triple.

d. $5^2 = 25 \quad 12^2 = 144 \quad 13^2 = 169$

$169 = 25 + 144$

Yes, 5, 12, and 13 is a Pythagorean triple.

e. $7^2 = 49 \quad 20^2 = 400 \quad 21^2 = 441$

$441 \neq 49 + 400$

No, 7, 20, and 21 is **not** a Pythagorean triple.

f. $1^2 = 1 \quad 10^2 = 100 \quad 11^2 = 121$

$121 \neq 1 + 100$

No, 1, 10, and 11 is **not** a Pythagorean triple.

10. **a.** $6^2 = 36$ $8^2 = 64$ $10^2 = 100$

$100 = 36 + 64$

Yes, 6, 8, and 10 is a Pythagorean triple.

b. $9^2 = 81$ $12^2 = 144$ $15^2 = 225$

$225 = 81 + 144$

Yes, 9, 12, and 15 is a Pythagorean triple.

c.

$$\begin{array}{ccc} 3, & 4, & 5 \\ \times 4 & \times 4 & \times 4 \\ \downarrow & \downarrow & \downarrow \\ = 12, & 16, & 20 \end{array} \quad \textbf{or} \quad \begin{array}{ccc} 3, & 4, & 5 \\ \times 5 & \times 5 & \times 5 \\ \downarrow & \downarrow & \downarrow \\ = 15, & 20, & 25 \end{array} \quad \textbf{or} \quad \begin{array}{ccc} 3, & 4, & 5 \\ \times 6 & \times 6 & \times 6 \\ \downarrow & \downarrow & \downarrow \\ = 18, & 24, & 30 \end{array}$$

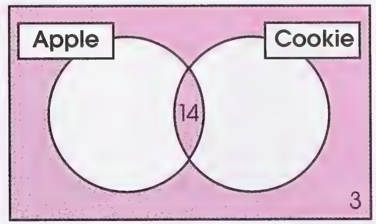
Other Pythagorean triples are 12, 16, and 20 **or** 15, 20, and 25, **or** 18, 24, and 30. Many other answers are possible.

11. Once you know a Pythagorean triple, you can find another by multiplying each number in the triple by the same number.

Now Try This

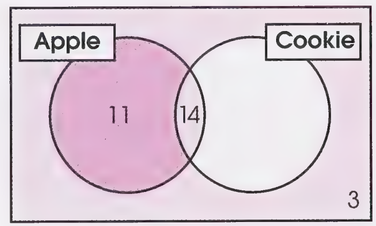
12. Use a Venn diagram to solve this problem.

Make a Venn diagram. Let one circle represent the number of students who brought an apple. Let a second circle represent the number of students who brought a cookie. The intersection of the circles represents the number of students who brought both an apple and a cookie. The area outside the circle represents the number of students who brought neither an apple nor a cookie.



Calculate how many students brought only an apple.

$25 - 14 = 11$

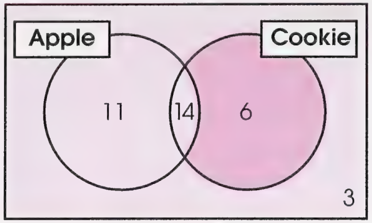


So, 11 students brought only an apple.

Calculate how many students brought only a cookie.

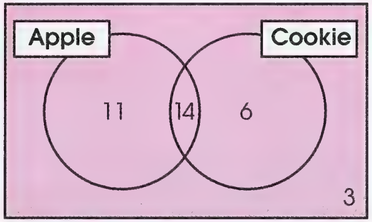
$20 - 14 = 6$

So, 6 students brought only a cookie.



Calculate the total number of students.

$11 + 14 + 6 + 3 = 34$



There are 34 students altogether.

Section 2: Follow-up Activities

Extra Help

1. **a.** yes
   b. no
   c. yes

2. **a.**

First Factor	a number between 1 and 10	8.93

893.

Second Factor	a power of ten	10^2

$\underbrace{893}_{\text{2 places}}.$

$\therefore\ 893 = 8.93 \times 10^2$

b.

First Factor	a number between 1 and 10	8.93

8 930 000.

Second Factor	a power of ten	10^6

$8\,\underbrace{930\,000}_{\text{6 places}}.$

$\therefore\ 8\,930\,000 = 8.93 \times 10^6$

c.

First Factor	a number between 1 and 10	8.93 — $8_\wedge 930.$
Second Factor	a power of ten	10^3 — $8_\wedge\underbrace{930}_{\text{3 places}}.$

$\therefore\ 8930 = 8.93 \times 10^3$

3. **a.** no **b.** no **c.** no

4. **a.**

First Factor	a number between 1 and 10	8.93 — $0.008_\wedge 93$
Second Factor	a power of ten	10^{-3} — $.\underbrace{008}_{\text{3 places}}{}_\wedge 93$

$\therefore\ 0.008\,93 = 8.93 \times 10^{-3}$

b.

First Factor	a number between 1 and 10	8.93 — $0.000\,8_\wedge 93$
Second Factor	a power of ten	10^{-4} — $0.\underbrace{000\,8}_{\text{4 places}}{}_\wedge 93$

$\therefore\ 0.000\,893 = 8.93 \times 10^{-4}$

c.

First Factor	a number between 1 and 10	8.93 — $0.000\,08_\wedge 9\,3$
Second Factor	a power of ten	10^{-5} — $0.\underbrace{000\,08}_{\text{5 places}}{}_\wedge 9\,3$

$\therefore\ 0.000\,089\,3 = 8.93 \times 10^{-5}$

5. Why couldn't Orgo keep his waterbed a secret?

BECAUSEITLEAKEDOUT (because it leaked out)

Enrichment

1. Answers may vary. The name probably ends in *-illion* because the names of most large numbers have this ending. The name may begin with the letter *z* because *z* is the last letter of the alphabet and this suggests that it is the largest. It also could end in *z* to contrast with zero, the smallest whole number.

2. **a.** $10\ 000\ 000 = 10^7$

   b. $100\ 000\ 000\ 000 = 10^{11}$

   c. $100\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000 = 10^{29}$

   d. $10\ 000\ 000\ 000\ 000 = 10^{13}$

3. No, Shawn's statement was not correct. The number 10^{18} or 1 000 000 000 000 000 000 is an estimate. The zeros simply act as place holders to show that the number has been rounded to the nearest quintillion. It is not reasonable to subtract 1 from such an inaccurate number.

4. **a.** $596\ 147\ 559\ 260.89 \doteq 596\ 000\ 000\ 000$
   On July 8,1997, Canada's national debt was about $596 000 000 000.

   b. The amount $596 000 000 000 is read as "five hundred ninety-six billion dollars."

   c. $596\ 000\ 000\ 000 = 5.96 \times 10^{11}$

Magic Clay Turned Bulb into Bauble

Gold and silver are putty in the hands of Matthew Todhunter, who has fashioned a life-size garlic bulb of gold and silver from precious-metal clay.

"I moulded the clay, containing pure silver particles, around an actual garlic bulb," explains the designer goldsmith at Alberta Gem Labs. "Then I added the stem on top from 24-carat gold."

The clay is about 75 percent precious metal and 25 percent binder.

When Todhunter fired his creation in a kiln, at 900 degrees Celsius for two hours, the binder vaporized. So did the real garlic bulb—leaving pure gold and silver ready for final detailing.

The bulb was inspired by the Sorrento and Sorrentino Restaurants' annual garlic festival, says Darryl Arthurs, owner of Alberta Gem Lab. The piece is one-fifth gold, four-fifths silver, weighs $3\frac{3}{4}$ ounces and is priced at $2500.

"With gold, you have been limited to casting or fabrication from sheet, or plating," Arthurs says.

"Now, you can virtually do any shape or form. You could imprint a leaf on the outside of something, and leave an


[2]

impression. You could never do that with molten material."

A creation could be pure gold, pure silver, any blend of the two metals or silver with gold highlights.[1]

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[1] Ron Chalmers, "Magic Clay Turned Bulb into Bauble," *The Edmonton Journal*, 15 April 1997, E1. Reprinted by permission.
[2] Jennifer Parker, photo, *The Edmonton Journal*, 15 April 1997, E1. Reprinted by permission.

Is Bailey the Fastest? Of Course He Is

Who is the fastest man in the world?

Is it Canada's Donovan Bailey who sprinted 100 metres in 9.84 seconds to set a world record?

Or is it American Michael Johnson who covered 200 m in 19.32 seconds to capture his own world record?

NBC announcer Bob Costas simply divided the 19.32 seconds by two to come out with a time of 9.66 seconds per 100 m.

Therefore, Johnson is the fastest, right?

Sounds reasonable until you talk to somebody who knows something about it.

"The fastest man is considered to be the 100-metre sprinter, always," says University of Alberta track coach Marek Glowacki.

Tradition aside, why is that?

Because simply dividing the 200 m time by two, doesn't fairly take into account the time for acceleration from the blocks needed to reach top speed.

"In the first hundred metres the runner is losing regularly about 65 to 80 hundreths of a second because they have to move from the standing position to full speed," said Glowacki.

"And the second hundred metres of the 200 metres is on the fly."

It's now been reported out of Atlanta that Johnson covered his first 100 m in 10:12 seconds and the second 100 in 9:20.

"In the first 100 metres, he ran 10:12 which is a good time, but Donovan Bailey is much faster than he is."

Furthermore, if you take Johnson's 9:20 in the second 100 m and add the usual 80/100ths of a second required when a top athlete starts, it comes out to an even 10 seconds, said Glowacki.

Still slower than Bailey.

The fastest man in the world?

A Canadian, Mr. Costas.[1]

Pickles Cartoon

[2]

[1] Cam Cole, "Is Bailey the Fastest? Of Course He Is," *The Edmonton Journal*, 3 August 1996, Cl. Reprinted by permission.
[2] Brian Crane, cartoon, "Pickles" © 1996, Washington Post Writers Group. Reprinted with permission.

The IMAX System

Date: 1970

Innovators: Filmmakers Graeme Ferguson, Robert Kerr, and Roman Kroitor, and engineer William Shaw, all of Toronto.

Significance: IMAX multiscreen films are a huge commercial and popular success. The IMAX system presents spectacularly vivid motion pictures on giant screens.

Expo origins: Among the most popular attractions at Expo 67 in Montreal were huge, multiple-screen films which created a vivid illusion of reality and left their audiences stunned at the power of the images. Roman Kroitor of the National Film Board co-produced one of these giant-screen films, while his friends Graeme Ferguson and Robert Kerr produced another. All three wanted to continue making giant-screen films, but not with the cumbersome systems of multiple projectors they had employed at Expo. They formed a company, Multiscreen Corporation, which won a contract to make a film for the Fuji pavilion at Osaka's 1970 Exposition, and they set out to develop a new system, one which would bathe audiences in life-like sound and fill the largest screens in cinema history with clear, bright, colourful images from a single projector.

Wide-screen Films: The three Canadian filmmakers hoped to use the widest conventional film in existence, 70-mm film, increasing the size and hence the visual impact, of the projected images by increasing the size of the film frames. Experts they consulted said this was technically impossible; a giant-frame film would have to be pulled through a projector as such speed that it would tear. The experts, though, were thinking of film moving vertically through the projector in the conventional way, one frame on top of the next. They were proved wrong by an Australian, Ron Jones, who in 1967 invented a mechanism which gently and rapidly moves film horizontally in rolling loops or waves, somewhat like a caterpillar. Using this mechanism, 70-mm film can run sideways, one frame beside the next, and these frames can be three times larger in area than conventional, 70-mm ones.

Recognizing this invention as what they were looking for, the three friends bought the rights to use it, and recruited engineer Bill Shaw, who went to high school in Galt, Ont., with Kerr and Ferguson, to build the prototype projector. Shaw knew nothing about projectors when he began developing the IMAX ("i" for image, "max" for maximum) projector. He learned quickly. With more than one million dollars borrowed from the federal government and from the young company's first client, Fuji Group, he developed something that sounded like a B-52 bomber when first started up, but that projected brilliant, giant images. It was ready just in time tor the Osaka exposition.

Using a camera built by a Norwegian designer in only four months, Roman Kroitor produced a film for Osaka that was a big hit. A year later, IMAX Systems Corporation, as the four founders renamed their company, equipped the first permanent IMAX theatre, at the Ontario Place theme park on the Toronto waterfront. In 1973, they built a second permanent theatre, this time for a San Diego planetarium. As an additional feature, Shaw put fisheye lenses on the IMAX camera and projector so that film could be projected onto a vast, domed screen, creating what was called the OMNIMAX system.

There are now IMAX or OMNIMAX theatres scattered around the world.[1]

[1] Sean McCutcheon, "Discoveries and Inventions," *Horizon Canada*, vol. 10, no. 113 (1987): third cover. Reprinted by permission.

The First Practical Electron Microscope

Date: Late 1930s

Innovators: Eli Franklin Burton, professor of physics, born at Green River, Ont., in 1879, died at Toronto in 1948; with **James Hillier** and **Albert Prebus**, both graduate students working under Burton's supervision.

Significance: With electron microscopes, scientists can now "see" things, such as viruses and molecules, which previously were invisibly small, thereby gaining insights into diseases, industrial materials and more.

Profile: Educated at the University of Toronto and Cambridge University, Eli Burton was a talented and imaginative professor who had already invented a device for electrically measuring moisture in wheat and timber when be became head of the physics department at the University of Toronto in 1932. Burton's research interests included colloids—substances, such as smoke, composed of particles far too small to be visible to the naked eye. Many colloids are so small they remain invisible even to the best optical microscopes.

In 1935, Burton visited Ernst Ruska's Berlin laboratory to see the "ultramicroscope," the world's first electron microscope. On his return he enthusiastically set out to improve on the crude instrument he had seen, assigning Hillier and Prebus the task of devising ways of controlling electrons, machining precise parts and assembling an intricate machine. In June 1938, Burton and his students first tested their creation by photographing the edge of a razor blade. It looked sharp, even when viewed through an optical microscope, but the electron microscope showed it to be an enormous and jagged mountain range. Completed in 1939, this instrument was the first built in North America. Both Hillier and Prebus moved to the United States after graduating, Hillier to develop commercial electron microscopes at the Radio Corporation of America (RCA).

How Do Electron Microscopes Differ from Light Microscopes? In principle, an electron microscope is like an optical microscope, but with a beam of electrons focussed by electromagnets instead of a beam of light focussed and magnified by glass lenses. The key difference between the two kinds of microscopes is in their ability to produce separate images of objects that are very close together, an ability that depends on the wavelength of radiation used. The light microscope cannot resolve objects that are closer together than the closest spacing between light waves, around 10 000 to a centimetre. Because the effective wavelengths of speeding electrons is so much smaller that the wavelengths of light, the electron microscope can resolve particles so small that more than 10 million of them, side by side, span one centimetre. The electron microscope is to the light microscope what surgical gloves are to boxing gloves: a finer probe. It can magnify objects as much as one million times without losing clarity.

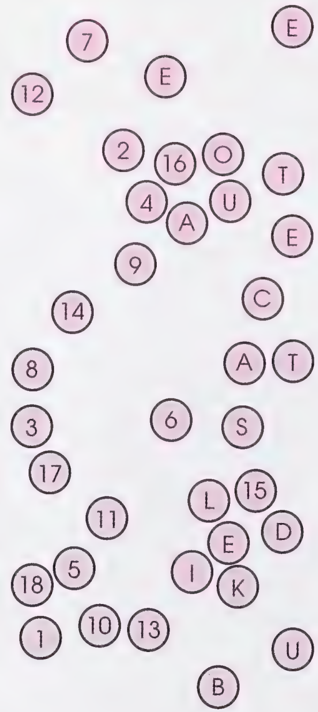
Did You Know? The electrons bombarding a sample in an electron microscope deliver the same amount of energy to unit area as did the atomic bomb that exploded over Hiroshima.[1]

[1] Sean McCutcheon, "Discoveries and Inventions," *Horizon Canada*, vol. 5, no. 60 (1986): third cover. Reprinted by permission.

Why Couldn't Orgo Keep His Waterbed a Secret?

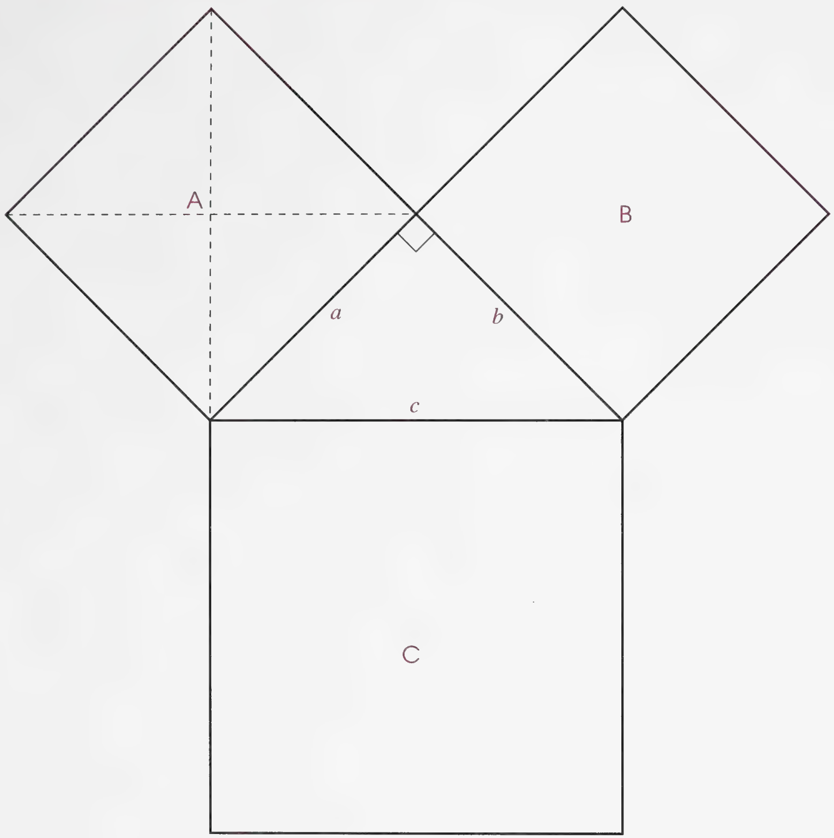
To answer this important question, express any number in the left column in scientific notation. Find your answer in the right column and draw a straight line connecting the two numbers. Each line will cross a number and a letter. The number tells you where to put the letter in the row of boxes at the bottom of the page.

7000 ■	■ 7×10^{7}
7 000 000 ■	■ 7×10^{-5}
70 ■	■ 7×10^{-4}
700 000 ■	■ 7×10^{2}
70 000 000 000 ■	■ 7×10^{3}
700 ■	■ 7×10^{-2}
70 000 000 ■	■ 7×10^{-3}
70 000 ■	■ 7×10^{5}
0.07 ■	■ 7×10^{-10}
0.000 007 ■	■ 7×10^{10}
0.007 ■	■ 7×10^{-6}
0.000 000 07 ■	■ 7×10^{6}
0.7 ■	■ 7×10^{-12}
0.000 07 ■	■ 7×10^{-8}
0.000 000 7 ■	■ 7×10^{1}
0.0007 ■	■ 7×10^{-1}
0.000 000 000 007 ■	■ 7×10^{-7}
0.000 000 000 7 ■	■ 7×10^{4}

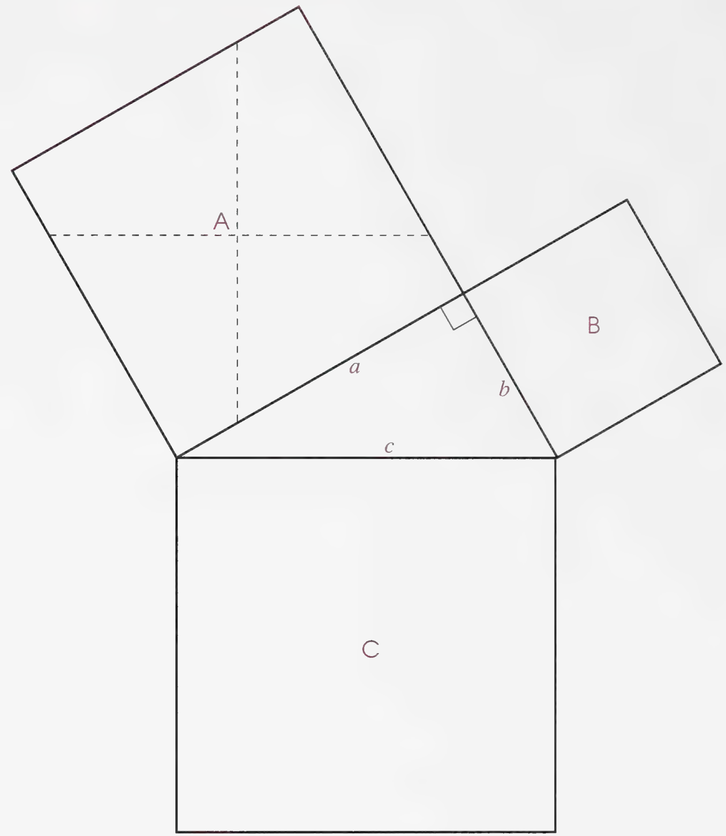


1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18

Pythagorean Puzzle 1



Pythagorean Puzzle 2



Student and teacher: Use this cover sheet for mailing or faxing.

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8110 Mathematics 8
Module 3

FOR STUDENT USE ONLY

Date Module Submitted: ____________

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(If label is missing or incorrect)

File Number: ____________

Module Number: ____________

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Assigned Teacher: ____________

Module Grading: ____________

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Student's Questions and Comments

Apply Module Label Here

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Please verify that preprinted label is for correct course and module.

Teacher's Comments

____

Teacher

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- Has your work been reread to ensure accuracy in spelling and details?
- Is the booklet cover filled out and the correct module label attached?

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Mathematics 8
Module 3

Number Applications

Assignment Booklet

Distance Learning

Alberta
EDUCATION

FOR TEACHER'S USE ONLY

Summary

	Total Possible Marks	Your Mark
Section 1 Assignment	65	
Section 2 Assignment	25	
Final Module Assignment	10	
	100	

Teacher's Comments

This document is intended for	
Students	✓
Teachers	✓
Administrators	
Parents	
General Public	
Other	

Mathematics 8
Assignment Booklet
Module 3
Number Connections
Learning Technologies Branch
ISBN 0-7741-1337-5

ASSIGNMENT BOOKLET
MATHEMATICS 8 – MODULE 3: NUMBER APPLICATIONS

Your mark on this module will be determined by how well you do your assignments in this booklet.

Work slowly and carefully. If you are having difficulties, go back and review the appropriate section.

There are two section assignments and one final module assignment in this Assignment Booklet. The total value of these assignments is 100 marks. The value of each assignment is stated in the left margin.

Be sure to proofread each assignment carefully.

65

Section 1 Assignment: Working with Proportional Situations

Read all the parts of your assignment carefully and record your answers in the appropriate place. Clearly show how you arrived at your answers.

3 **1.** Pancake batter may be made with pancake mix and water. The ratio of pancake mix to water is 25 to 16. If 320 mL of water is used, how much pancake mix is required?

2 **2.** A certain recipe calls for 250 mL of sugar, 500 mL of oatmeal, and 750 mL of flour. Write the amounts of the ingredients as a three-term ratio in lowest terms. **Hint:** Be sure to write the answer in a statement.

3. Following are the labels from four fertilizers. The labels indicate the percents of nitrogen, phosphorus, and potassium.

All-Purpose Fertilizer 20-20-20	**Garden Food Fertilizer** 6-12-12
Evergreen Tree Fertilizer 30-10-10	**Lawn Fertilizer** 10-6-4

1 **a.** Which fertilizer contains the same amount of nitrogen, phosphorus, and potassium?

1 **b.** Which fertilizer contains the greatest percent of nitrogen?

1 **c.** Which fertilizer has half as much nitrogen as potassium?

1 **d.** Which fertilizer has the greatest ratio of nitrogen to potassium.

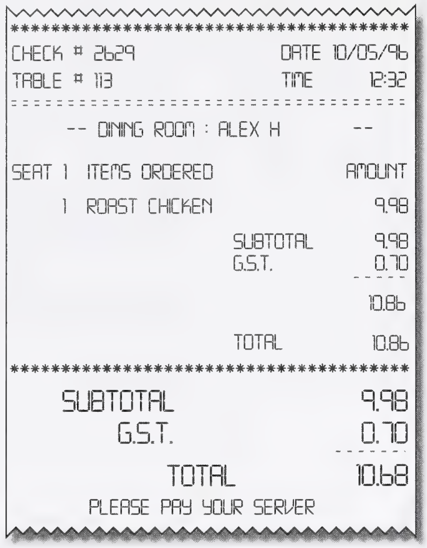
1 **e.** Which fertilizer has the greatest ratio of phosphorus to potassium?

(2) **4.** Ruth paid \$3.60 for 1.5 dozen apples. Write the statement as a rate in simplest form.

(3) **5.** Which is the best buy: \$2.99 for 400 g of soap, \$2.39 for 300 g of soap, or \$1.56 for 200 g of soap?

(3) **6.** June can walk 2 blocks in 5 min. At this rate, how far can she walk in 1 h?

3 **7.** Adelle had a meal at a restaurant. This was her bill.



CHECK # 2629		DATE	10/05/96
TABLE # 113		TIME	12:32

-- DINING ROOM : ALEX H --

SEAT 1	ITEMS ORDERED		AMOUNT
1	ROAST CHICKEN		9.98
		SUBTOTAL	9.98
		G.S.T.	0.70
			10.86
		TOTAL	10.86

SUBTOTAL	9.98
G.S.T.	0.70
TOTAL	10.68

PLEASE PAY YOUR SERVER

The service was good and it is customary to give a tip of about 15%. About how much money should Adelle give for a tip?

2 **8.** The price of a bicycle is \$320. If the goods and services tax (GST) is 7%, what is the total cost of the bicycle in Alberta? **Note:** There is no provincial sales tax (PST) in Alberta.

9. Calculate the simple interest in each of the following situations. **Hint:** $I = Prt$

3 **a.** \$3500 invested at 8.5%/a for 6 mo

3 **b.** \$4000 borrowed at 2.25%/mo for 1 a

10. A coat was regularly priced at $375. It was put on sale for 20% off. The sale price was then reduced by another 25%.

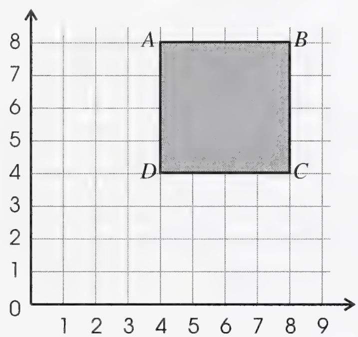
(2) a. What was the first sale price?

(2) b. What was the second sale price?

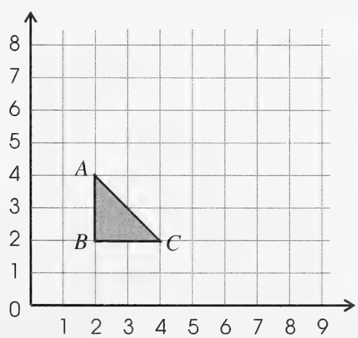
(3) 11. A scale model was made of a walking stick, the longest known insect in the world today. The scale was 60 to 1. If a walking stick can be 33 cm long, how long was the scale model? Give the length in metres.

5 **12.** In a newspaper, find a cartoon strip that you like. Draw a 0.5-cm grid on one of the panels of the cartoon strip. Then use 1-cm graph paper to make an enlargement of the panel. **Note:** Include the original cartoon strip that you used and the enlargement. Be sure to clearly indicate the following information on both the cartoon strip and the enlargement: your name, registration number or class (if applicable), the subject, and the module number.

2 **13. a.** Draw an image of square *ABCD* using a scale factor of 0.25. The dilatation centre is $(0,0)$.



2 **b.** Draw an image of triangle *ABC* using a scale factor of 1.5. The dilatation centre is $(0,0)$.



1 **c.** If you connected the dilatation centre and a pair corresponding vertices in questions 13.a. or 13.b., what would you notice?

____

____

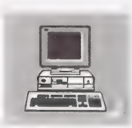
Use the following map to answer question 14.

1 **14. a.** What does 1 cm represent on the map?

____

____

3 **b.** What is the actual straight-line distance (in kilometres) from the northern tip of Brazil to the southern tip of Brazil?



You may do question 15 using the graph paper provided or you may use a computer and spreadsheet program. **Note:** If you use a computer, print the spreadsheet and the graph and include them with this assignment. Be sure to clearly indicate your name, registration number or class (if applicable), the subject, and the module number on the printout.

15. A liquid is boiling and the amount of liquid left after each hour is measured. Following is a table showing the amount of liquid (in litres) left after each hour.

Amount of Time (in hours)	Amount Left (in litres)
0	7
1	5
2	3.8
3	2.5
4	2
5	1.5
6	1.2
7	0.8

4 **a.** Use the given data to make a graph.

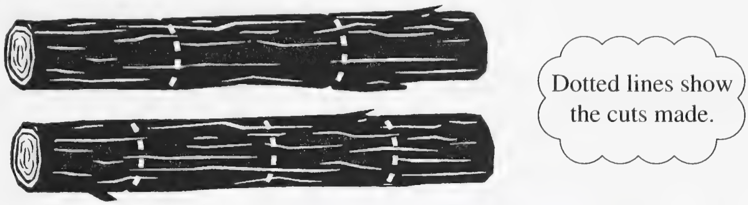
2 **b.** Is this a proportional situation? Why or why not?

16. It is 10 km to the airport from the hotel. The speed limit is 110 km/h. Marika has only 9 min to get to the airport. The traffic is heavy, so Marika travels only 50 km/h for the first 5 km.

3 **a.** How long will it take Marika to go the first 5 km? Express the answer in minutes.
Hint: $d = st$.

3 **b.** At what speed must Marika travel the rest of the way to get to the airport on time?

3 **17.** It takes Frank 4 min to saw a log into three pieces. At this rate, how long will it take Frank to saw a similar log into four pieces? Assume that no time is wasted between sawing one cut and another. **Note:** Be careful to use the number of cuts, not the number of pieces.



25

Section 2 Assignment: Working with Powers and Roots

Read all the parts of your assignment carefully and record your answers in the appropriate place. Clearly show how you arrived at your answers.

1. Write each of the following statements in standard form.

2 **a.** A typical bed usually has over 6×10^9 dust mites.

____

____

2 **b.** The diameter of a human hair is 7×10^{-3} cm.

____

____

2. Write each of the following statements in scientific notation.

2 **a.** The mass of one molecule of water is 0.000 000 000 000 031 mg.

____

____

2

b. The distance from the Sun to Neptune is 478 000 000 km.

____

____

3

3. A student understands how to model and perform operations with integers, but does not understand the concept of square root.

To help the student understand what $\sqrt{16}$ means, write an explanation and use diagrams of counters.

3

4. a. Explain why $\sqrt{50}$ is not a whole number. Use diagrams if you wish.

1 **b.** Approximate $\sqrt{50}$ using your calculator. Round to the nearest tenth.

5. Some clocks have pendulums that swing back and forth. Rhonda changed the length of the pendulum of a clock and measured the swing times. Her results are recorded in the following table.

Length of Pendulum (units)	Time of Swing (seconds)
1	1
4	2
9	3
16	4

2 **a.** From the pattern in the table, how does the swing time seem to relate to the length of the pendulum?

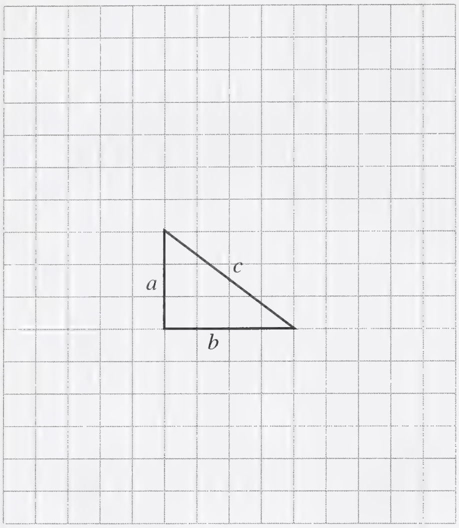
____

____

____

2 **b.** What do you think the swing time would be of a pendulum with a length of 36 units?

3 **6.** A student does not know the Pythagorean relation. Use the given graph paper and explain to the student why, in the given right triangle, $a^2 + b^2 = c^2$.



3 **7.** Jamie wants to walk from one corner of the rectangular park to the diagonally opposite corner. The park is 30 m by 50 m. How far does she walk? Round the answer to the nearest tenth. **Hint:** Make a diagram.

10

Final Module Assignment

Read all the parts of your assignment carefully and record your answers in the appropriate place. Clearly show how you arrived at your answers.

Use the following cartoon to answer question 1.

2

1. If 98 000 Turkish lira are equivalent to C$1, is C$10 enough to make you a millionaire in Turkish lira? Explain. **Note:** C$1 is read as "one Canadian dollar."

Use the following information to answer question 2.

> Have you read or heard of the book by Jonathan Swift called *Gulliver's Travels*? Gulliver, a ship captain, suffers a shipwreck and finds himself in the land of Lilliput. Here he finds that the heights of the people, plants, and animals are in a 1:12 ratio to the heights of the people, plants, and animals in his world.

2 **2. a.** Use a metric ruler to measure the length of your middle finger.

My middle finger is about ____________ cm.

Next, calculate the approximate length of the middle finger of a Lilliputian (a person from Lilliput).

1 **b.** Each day the Emperor of Lilliput gave Gulliver the food and drink necessary to feed about 1728 Lilliputians. How did the Emperor's mathematicians arrive at this number? Explain why this should be about the right amount.

____

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____

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5

3. Use the following grid to make a scale drawing of the floor plan of your bedroom. **Note:** Be sure to indicate the scale.

ASSIGNMENT BOOKLET DECLARATIONS

The Student's Declaration is to be filled in by a student registered at the Alberta Distance Learning Centre. If the student is under 16, the Learning Facilitator's Declaration is to be filled in by the learning facilitator. Failure to complete this page may invalidate the assignment results.

STUDENT'S DECLARATION

- I have followed the instructions outlined in the Student Module Booklet.
- I have completed the activities to prepare myself for the assignments in this Assignment Booklet.
- I completed the assignments in this Assignment Booklet by myself.

____

Student's Signature

LEARNING FACILITATOR'S DECLARATION

I hereby certify that I have supervised the learning activities completed by ____________________.

Student's Name

I also certify that to the best of my knowledge the assignments in this Assignment Booklet were completed independently by this student.

____

Supervisor's Signature

If you, the student or learning facilitator, have any comments or observations regarding this module, write them in the following space.

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